1 Problem 1:

1. $mr_1v_1 = mr_2v_2$, so $v_2 = v_1 \cdot \frac{r_1}{r_2}$.
   2. $W = E_{k2} - E_{k1} = \frac{1}{2}m\left(v_1^2 \cdot \frac{r_1^2}{r_2^2}\right) - \frac{1}{2}mv_1^2 = W = \frac{1}{2}mv_1^2\left(\frac{r_1^2}{r_2^2} - 1\right)$
   3. $m\frac{\left(\frac{v_1r_1}{r}\right)^2}{r} = m\frac{v_1^2r_1^2}{r^3}$

2 Problem 2:

1. We have $F\cos\alpha = Ma + f$, thus $f = F\cos\alpha - Ma$.
   So take it to np slipping rotation: $F \cdot r - f \cdot R = I \cdot \frac{a}{R}$,

$$\begin{aligned} Fr - FR\cos\alpha + MaR &= \frac{Ia}{R} \\ F(r - R\cos\alpha) &= a\left(\frac{I}{R} - MR\right) \\ a &= \frac{FR(r - R\cos\alpha)}{I - MR^2} \end{aligned}$$

2. $F = \frac{Mg}{\sin\alpha}$.
   3.

$$\begin{aligned} \mu &\geq \frac{f}{Mg - Fsin\alpha} \\ &\geq \frac{F\cos\alpha - M\frac{FR(r-R\cos\alpha)}{I-MR^2}}{Mg - Fsin\alpha} \\ &\geq F\frac{\cos\alpha(I - MR^2) - MR(r - R\cos\alpha)}{(I - MR^2)(Mg - Fsin\alpha)} \\ &\geq F\frac{\cos\alpha I - MRr}{(I - MR^2)(Mg - Fsin\alpha)} \end{aligned}$$

When $cos\alpha = MRr/I$.

3 Problem 3:

We have $I = m(v_2 + v_1)$ and $I(h - r) = \frac{2}{5}mr^2\left(\frac{v_2}{r} + \frac{v_1}{r}\right) = \frac{2}{5}mr(v_2 + v_1)$, so we get $m(v_2 + v_1)(h - r) = \frac{2}{5}mr(v_2 + v_1)$, hence h = 7/5r.

If you write in other way you would get h=3/5r.

4 Problem 4:

1.

$$\begin{cases} \lambda_1\dot{\omega}_1 + (\lambda_3 - \lambda_2)\omega_2\omega_3 = 0 \\ \lambda_2\dot{\omega}_2 + (\lambda_1 - \lambda_3)\omega_1\omega_3 = 0 \\ \lambda_3\dot{\omega}_3 + (\lambda_2 - \lambda_1)\omega_1\omega_2 = 0 \end{cases} \tag{1}$$

Since $L_1 = \lambda_1\omega_1, \quad L_2 = \lambda_2\omega_2, \quad L_3 = \lambda_3\omega_3$, and $L^2 = L_1^2 + L_2^2 + L_3^2 = \lambda_1^2\omega_1^2 + \lambda_2^2\omega_2^2 + \lambda_3^2\omega_3^2$, we have

$$\begin{aligned}
\frac{d(L^2)}{dt} &= 2\lambda_1^2\omega_1\dot{\omega}_1 + 2\lambda_2^2\omega_2\dot{\omega}_2 + 2\lambda_3^2\omega_3\dot{\omega}_3 \\
&= 2\lambda_1^2\omega_1 \cdot \frac{(\lambda_2-\lambda_3)\omega_2\omega_3}{\lambda_1} + 2\lambda_2^2\omega_2 \cdot \frac{(\lambda_3-\lambda_1)\omega_1\omega_3}{\lambda_2} + 2\lambda_3^2\omega_3 \cdot \frac{(\lambda_1-\lambda_2)\omega_1\omega_2}{\lambda_3} \\
&= 2\lambda_1\omega_1\omega_2\omega_3(\lambda_2-\lambda_3) + 2\lambda_2\omega_1\omega_2\omega_3(\lambda_3-\lambda_1) + 2\lambda_3\omega_1\omega_2\omega_3(\lambda_1-\lambda_2) \\
&= 2\omega_1\omega_2\omega_3\left[\lambda_1(\lambda_2-\lambda_3) + \lambda_2(\lambda_3-\lambda_1) + \lambda_3(\lambda_1-\lambda_2)\right] \\
&= 2\omega_1\omega_2\omega_3 \cdot 0 \\
&= 0
\end{aligned} \tag{2}$$

Or, $\frac{d\vec{L}}{dt} = \tau - \vec{\omega} \times \vec{L} = \vec{\omega} \times \vec{L}$, so we have $\frac{d\vec{L}^2}{dt} = 2\vec{L}\frac{d\vec{L}}{dt} = 2\vec{L}\vec{\omega} \times \vec{L} = 0$.

2. For $T = \frac{1}{2}(\lambda_1\omega_1^2 + \lambda_2\omega_2^2 + \lambda_3\omega_3^2)$, we have

$$\begin{aligned}
\frac{dT}{dt} &= \lambda_1\omega_1\dot{\omega}_1 + \lambda_2\omega_2\dot{\omega}_2 + \lambda_3\omega_3\dot{\omega}_3 \\
&= \lambda_1\omega_1 \cdot \frac{(\lambda_2-\lambda_3)\omega_2\omega_3}{\lambda_1} + \lambda_2\omega_2 \cdot \frac{(\lambda_3-\lambda_1)\omega_1\omega_3}{\lambda_2} + \lambda_3\omega_3 \cdot \frac{(\lambda_1-\lambda_2)\omega_1\omega_2}{\lambda_3} \\
&= \omega_1\omega_2\omega_3(\lambda_2-\lambda_3) + \omega_1\omega_2\omega_3(\lambda_3-\lambda_1) + \omega_1\omega_2\omega_3(\lambda_1-\lambda_2) \\
&= \omega_1\omega_2\omega_3 \cdot 0 \\
&= 0
\end{aligned} \tag{3}$$