2. $E_K: \sum_i^N \frac{1}{2} m_i v_i^2$ $\quad E_G: \sum_i^N m_i g z_i$ $\quad E_U = \sum_{0 \le i < j \le N} U(|\vec{r_i} - \vec{r_j}|) = \frac{1}{2}\sum_{i \ne j} U(|\vec{r_i} - \vec{r_j}|)$

$$E_{tot} = E_K + E_G + E_U = \sum_i^N \left(\frac{v_i^2}{2} + g z_i\right) m_i + \frac{1}{2}\sum_{i \ne j} U(|\vec{r_i} - \vec{r_j}|)$$

$\frac{d}{dt} E_K = \vec{v_i} \cdot \vec{F_i}^{tot} = \vec{v_i}(\vec{F}_i^{int} + \vec{F}_i^{ext})$ $\quad \frac{d}{dt} E_G = m g \dot{z}_i = -\vec{v_i} \cdot \vec{F_i}^{ext}$ $\quad \frac{d}{dt} E_U = \frac{1}{2}\sum_{i \ne j} \nabla_i U_{ij} \cdot \vec{v_i} = \sum_i \vec{v_i} \cdot \vec{F_i}^{int} \Rightarrow \frac{d}{dt} E_{tot} = 0$

3.

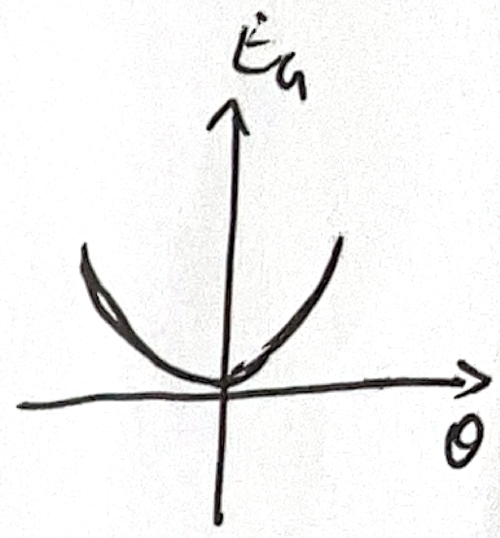
$y_1 = r\cos\theta$ $\quad y_2 = r\sin^2\theta \sim r\theta^2$ $\quad y_3 = b\cos\theta$

$y' = y_1 + y_2 + y_3 = (r+b)\cos\theta + r\theta^2$ $\qquad y = r + b$ for $\theta = 0$.

for small θ $\quad \simeq (r+b)\left(1 - \frac{\theta^2}{2}\right) + r\theta^2$

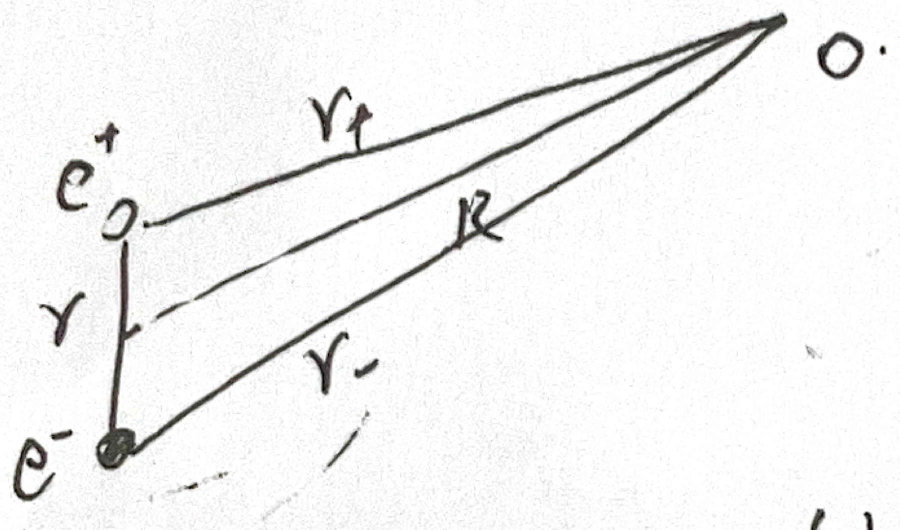
$= r + b + (r-b)\frac{\theta^2}{2}$

$\Delta y = y' - y = (r-b)\frac{\theta^2}{2}$



when $\Delta y > 0$, $mg\Delta y > 0$, cube is stable.

4.



$r_+ = \left(R^2 + \frac{r^2}{2} - Rr\cos\theta\right)^{1/2} \sim \left(R^2 - Rr\cos\theta\right)^{1/2}$

$r_- = \left(R^2 + \frac{r^2}{2} + Rr\cos\theta\right)^{1/2} \sim \left(R^2 + Rr\cos\theta\right)^{1/2}$

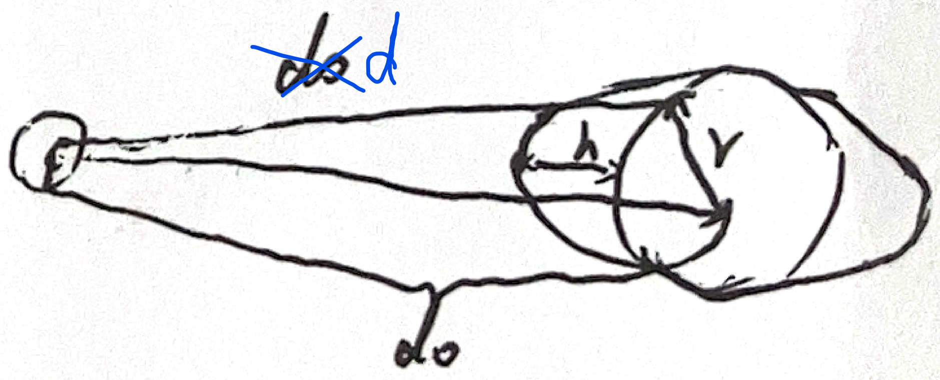
$$U_{(r)} = \frac{e}{4\pi\varepsilon_0 r_+} + \frac{-e}{4\pi\varepsilon_0 r_-}$$

$$= \frac{e}{4\pi\varepsilon_0} \cdot \frac{1}{R} \cdot \left[\left(1 - \frac{r}{R}\cos\theta\right)^{-1/2} - \left(1 + \frac{r}{R}\cos\theta\right)^{-1/2}\right]$$

$$= \frac{e}{4\pi\varepsilon_0} \cdot \frac{1}{R}\left(\frac{r}{2R}\cos\theta + \frac{r}{2R}\cos\theta\right)$$

$$= \frac{e}{4\pi\varepsilon_0} \cdot \frac{r\cos\theta}{R^2} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p} \cdot \vec{R}}{R^3} \qquad \vec{p} = \vec{r} \cdot e$$

5. $\vec{F}_{tide}(\vec{r}) = -GMm\left(\frac{\vec{d_0} + \vec{r}}{|\vec{d_0} + \vec{r}|^3} - \frac{\vec{d_0}}{d_0^3}\right)$



$$U = U_{moon} + U_{E_0} = -GM\left(\frac{1}{|\vec{d_0} + \vec{r}|} - \frac{1}{d_0} - \frac{\vec{d_0} \cdot \vec{r}}{d_0^3}\right)$$

$$-\nabla U = -GMm\left(\nabla \frac{1}{|\vec{d}|} - \nabla \frac{\vec{d_0} \cdot \vec{r}}{|\vec{d_0}|^3}\right)$$

$$= -GMm\left(\frac{\vec{d_0} + \vec{r}}{|\vec{d_0} + \vec{r}|^3} - \frac{\vec{d_0}}{d_0^3}\right) = \vec{F}_{tide}(\vec{r})$$

$$U = U_{tide} + U_{Earth} = -GM\left(\frac{1}{|\vec{d_0} + \vec{r}|} - \frac{1}{d_0} - \frac{\vec{d_0} \cdot \vec{r}}{d_0^3}\right) + mgh$$

$$\frac{1}{|\vec{d_0} + \vec{r}|} = \frac{1}{d_0} \frac{1}{|\hat{d_0} + \frac{\vec{r}}{d}|} = \frac{1}{d_0}\left[1 + \hat{d_0} \cdot \frac{\vec{r}}{d_0} - \frac{1}{2}\frac{r^2}{d_0^2} + \frac{3}{8} \cdot 4\left(\frac{\hat{d_0} \cdot \vec{r}}{d_0}\right)^2 + O\left(\frac{r}{d}\right)^3 \cdots\right]$$

$$U = -GMm\left[\frac{3(\hat{d_0} \cdot \vec{r})^2 - |\vec{r}|^2}{2|d_0|^3}\right] + mgh \begin{cases} r \parallel d_0 & -\frac{GMm r^2}{d_0^3} + mgh_{ebb} \\ r \perp d_0 & \frac{GMm r^2}{2 d_0^3} + mgh_{flood} \end{cases} \Rightarrow \Delta h = \frac{3GMr^2}{2g d_0^3}$$