

HW 2.

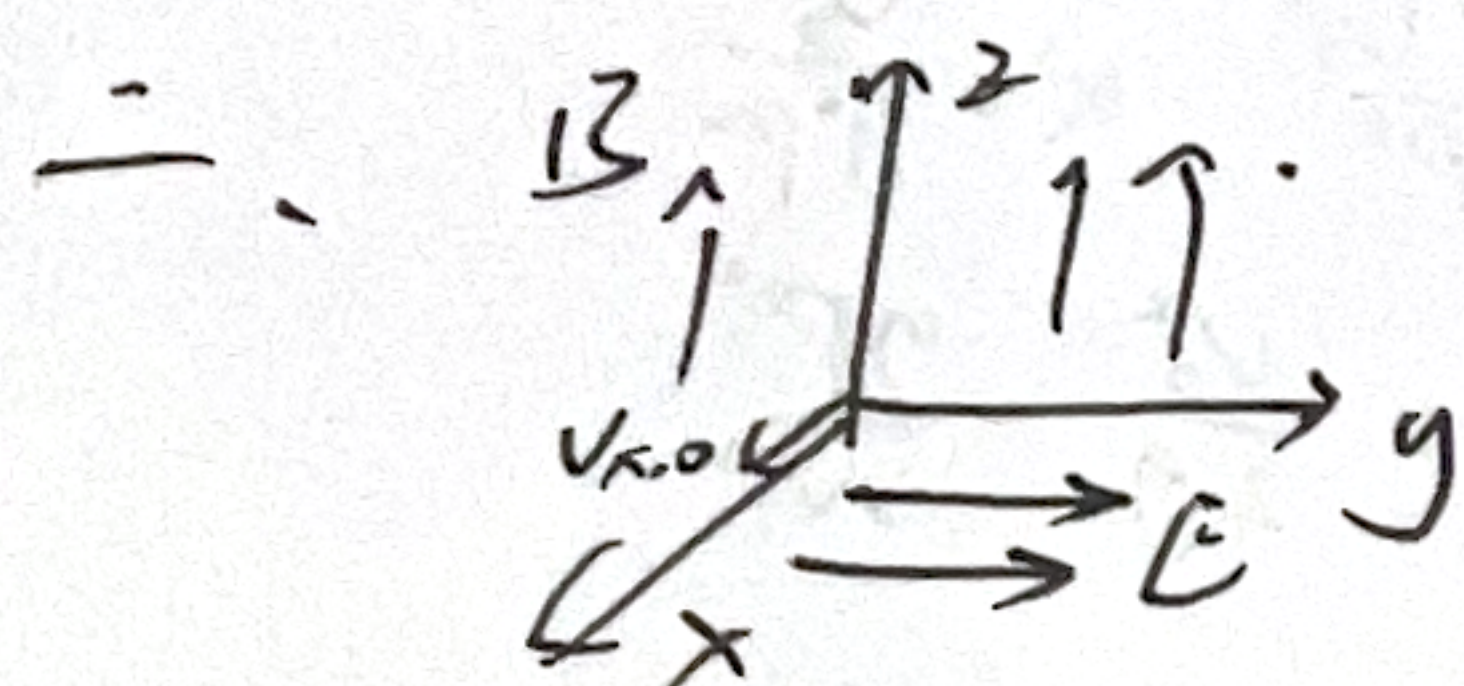
1.  $\dot{v} = \frac{dv}{dt} = \frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx}$

2.  $m\dot{v} = F(x)$

$$m v \frac{dv}{dx} = F(x) \Rightarrow \int_{v_0}^v \frac{m}{2} dv^2 = \int_{x_0}^x F(x') dx'$$

$$\Rightarrow v^2 = v_0^2 + \frac{2}{m} \int_{x_0}^x F(x') dx'$$

if  $F = F_0$ :  $v^2 = v_0^2 + \frac{2}{m} F_0 (x - x_0)$  /  $\Delta E_{kin} = F \cdot \Delta x$   
能量功



1.  $m\dot{\vec{v}} = \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$   $\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix}$

$\hat{x}$ :  $m\dot{v}_x = q(v_y B_z)$

$\hat{y}$ :  $m\dot{v}_y = q(E - v_x B_z)$

$\hat{z}$ :  $m\dot{v}_z = q(0 - (v_x B_y - v_y B_x)) = 0 \Rightarrow v_{z,0} = 0, \dot{v}_z = 0 \Rightarrow v_z(t) = 0$

so the motion remains in  $z=0$  plane.

2. ~~not~~ undeflected:  $\vec{F} = 0 \Rightarrow v = \frac{\vec{E} \times \vec{B}}{B^2} \Rightarrow v_{drift} = \frac{E}{B} \hat{x}$

3.  $m\dot{v}_x = q v_y B_0 \Rightarrow v_y = \frac{m}{qB} \dot{v}_x$   $\wedge m\dot{v}_y = q(E - B v_x)$

$$B \ddot{v}_x + \omega^2 v_x - \omega^2 v_{drift} = 0 \quad \omega = \frac{qB}{m}$$

$$v_x(t) = v_{drift} + A \cos(\omega t) + B \sin(\omega t)$$

$$(v_y = \frac{1}{\omega} \dot{v}_x) \quad v_y(t) = -A \sin(\omega t) + B \cos(\omega t)$$

since  $v_x(0) = v_{x0}, v_y(0) = 0$

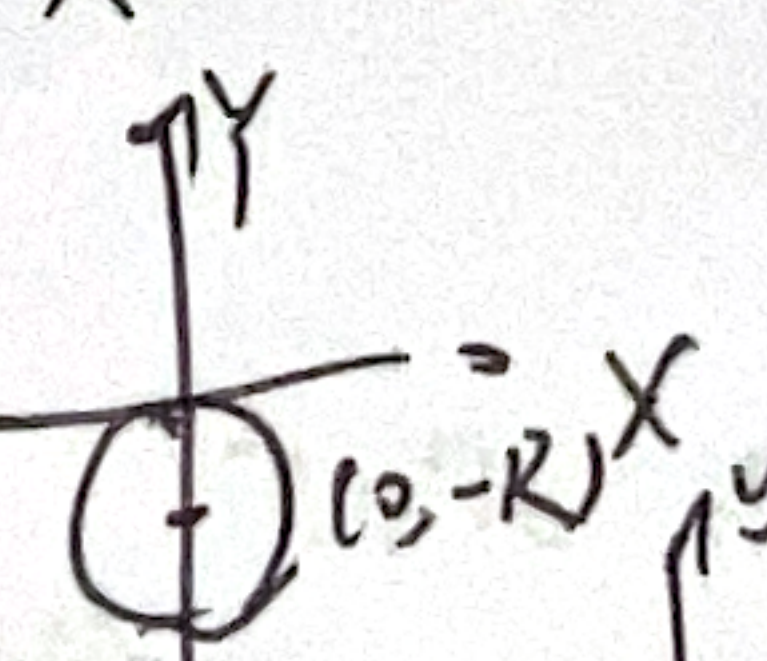
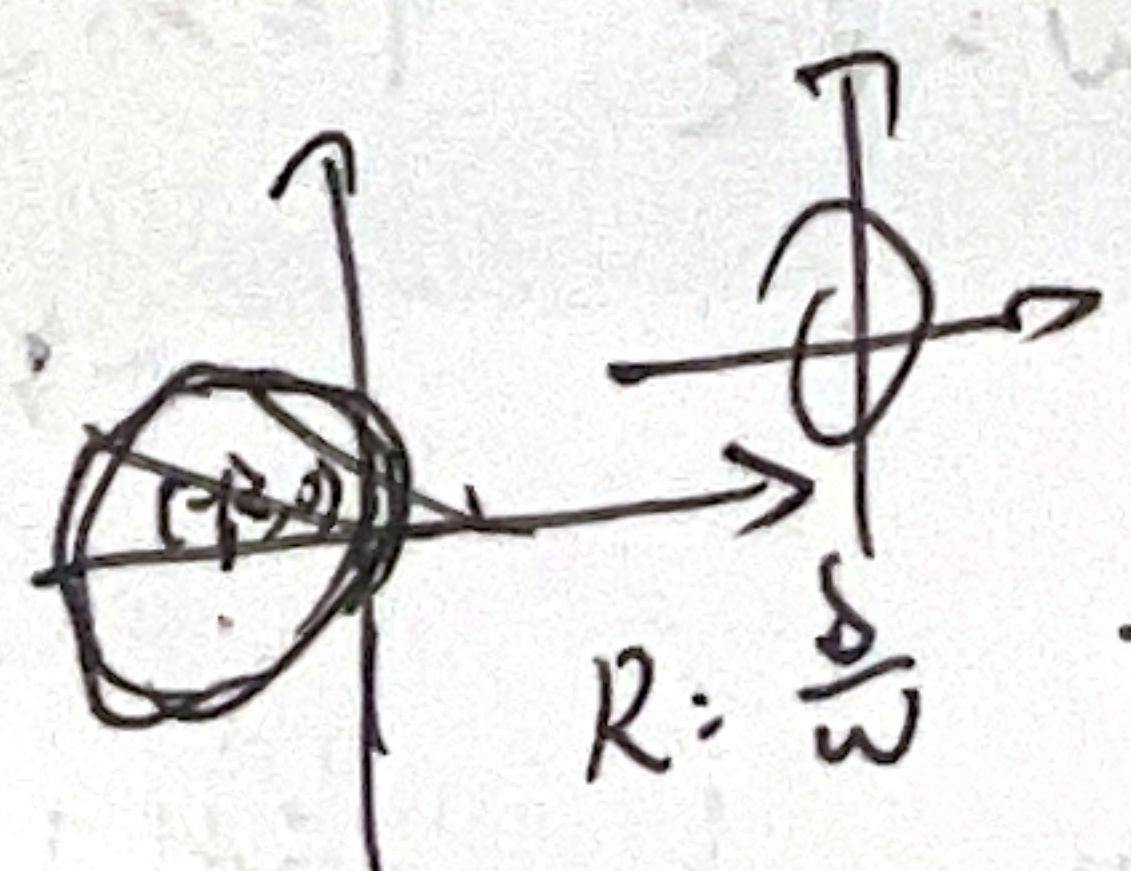
$$v_{x0} = v_{drift} + A, \quad 0 = -A \sin(0) + B \cos(0) \Rightarrow B = 0$$

$$x(t) = \int_0^t v_x(t') dt' = \frac{E}{B} t + (v_{x0} - \frac{E}{B}) \frac{\sin(\omega t)}{\omega}$$

$$y(t) = \int_0^t v_y(t') dt' = -\frac{(v_{x0} - \frac{E}{B})}{\omega} (1 - \cos(\omega t))$$

4.  $X(t) = x(t) - \frac{E}{B} t = \frac{E}{\omega B} \sin(\omega t)$   $Y(t) = y(t) = -\frac{E}{\omega B} (1 - \cos(\omega t))$

$$X^2 + (Y + \frac{E}{\omega B})^2 = (\frac{E}{\omega B})^2 \Rightarrow R = \frac{E}{\omega B}$$

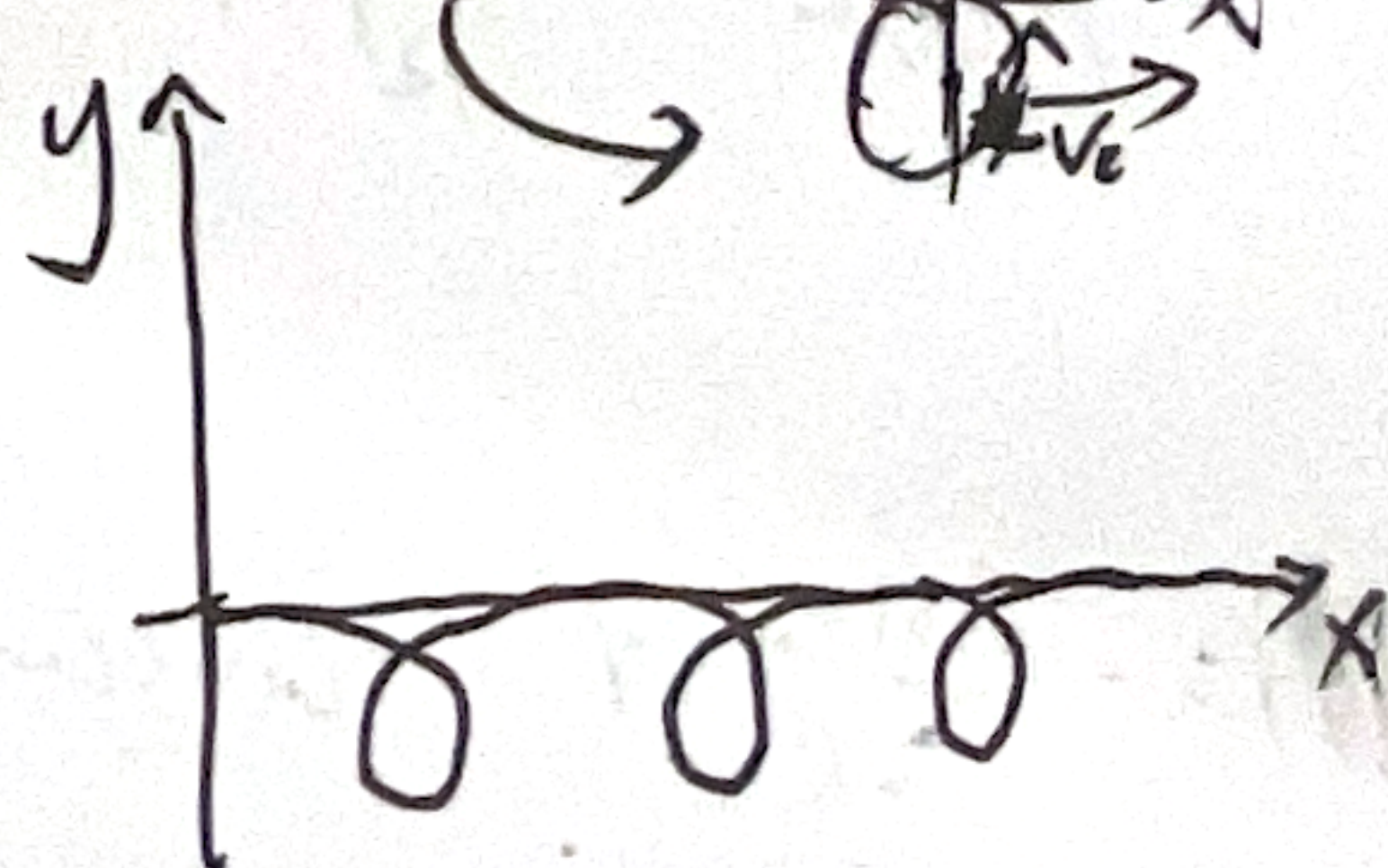


$\Rightarrow v_c = \frac{E}{B}$  平移的圆周运动

Circular motion ( $R = \frac{E}{\omega B}$ )

+ center translation ( $v_c = \frac{E}{B}$ )

or Trochoid/Cycloid





3:

Prove:  $z = \frac{v_0^2}{2g} - \frac{gp^2}{2v_0^2}$

$(x-y)p: v_0 \cos \theta \quad z: v_0 \sin \theta$

$p(t) = v_0 \cos \theta \cdot t$   
 $t = \frac{p}{v_0 \cos \theta}$

$z(t) = v_0 \sin \theta \cdot t - \frac{1}{2} g t^2$

$z = p \tan \theta - \frac{gp^2}{2v_0^2 \cos^2 \theta}$

$= p \tan \theta - \frac{gp^2}{2v_0^2} (1 + \tan^2 \theta)$

$= p x - g (1 + x^2)$

$\cos^2 \theta = \frac{1}{1 + \tan^2 \theta}$

find the maximum  $z(x)$ ,

$z_{max} = \frac{v_0^2}{g} - \frac{gp^2}{2v_0^2} \left( \frac{p^2 + v_0^2}{g^2 p^2} \right) = \frac{v_0^2}{g} - \frac{gp^2}{2v_0^2}$

$= \frac{v_0^2}{g} - \frac{gp^2}{2v_0^2} = \frac{v_0^2}{g} - \frac{gp^2}{2v_0^2}$

$p = 2gx \Rightarrow x = \frac{p}{2g}$

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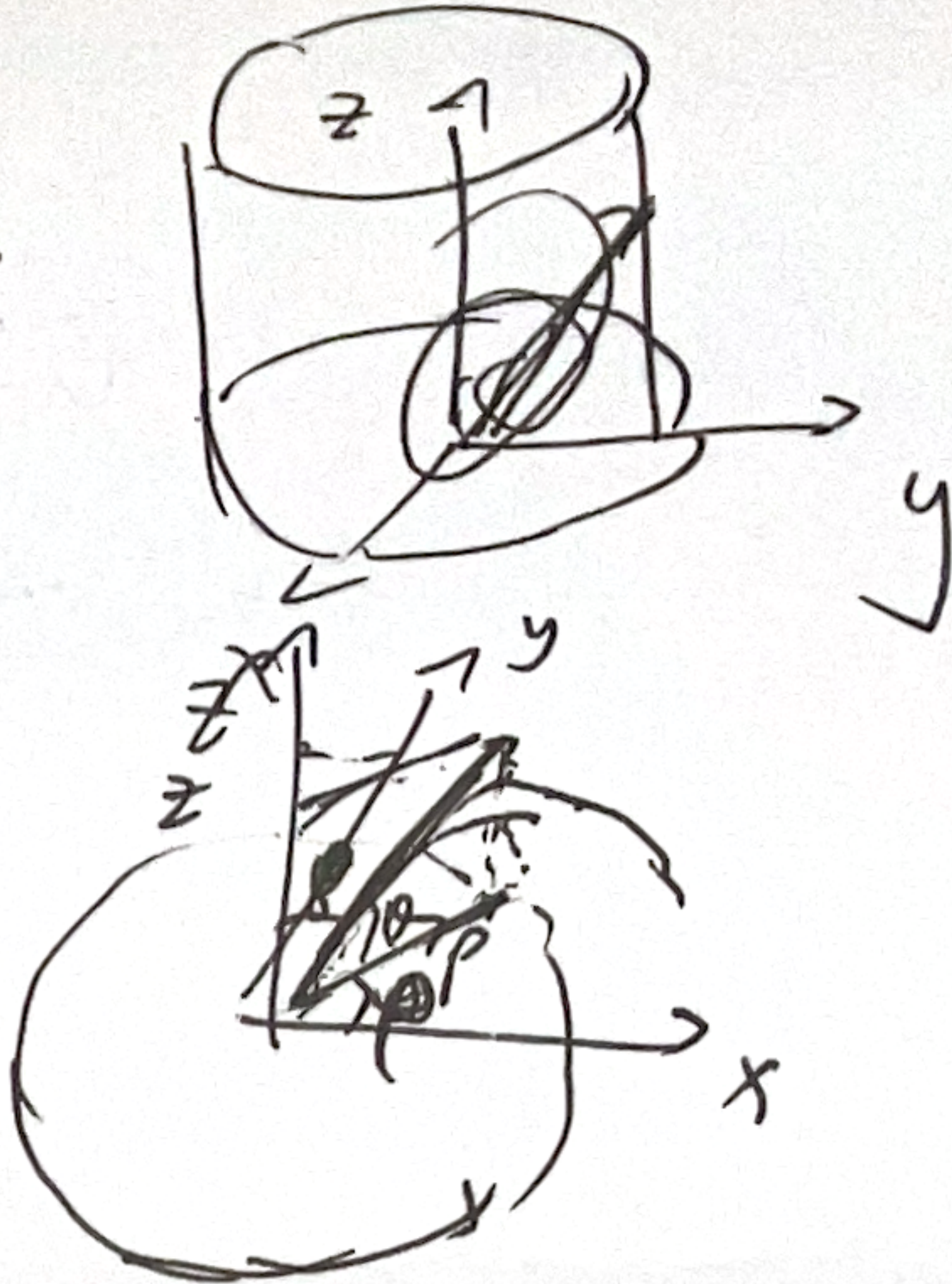
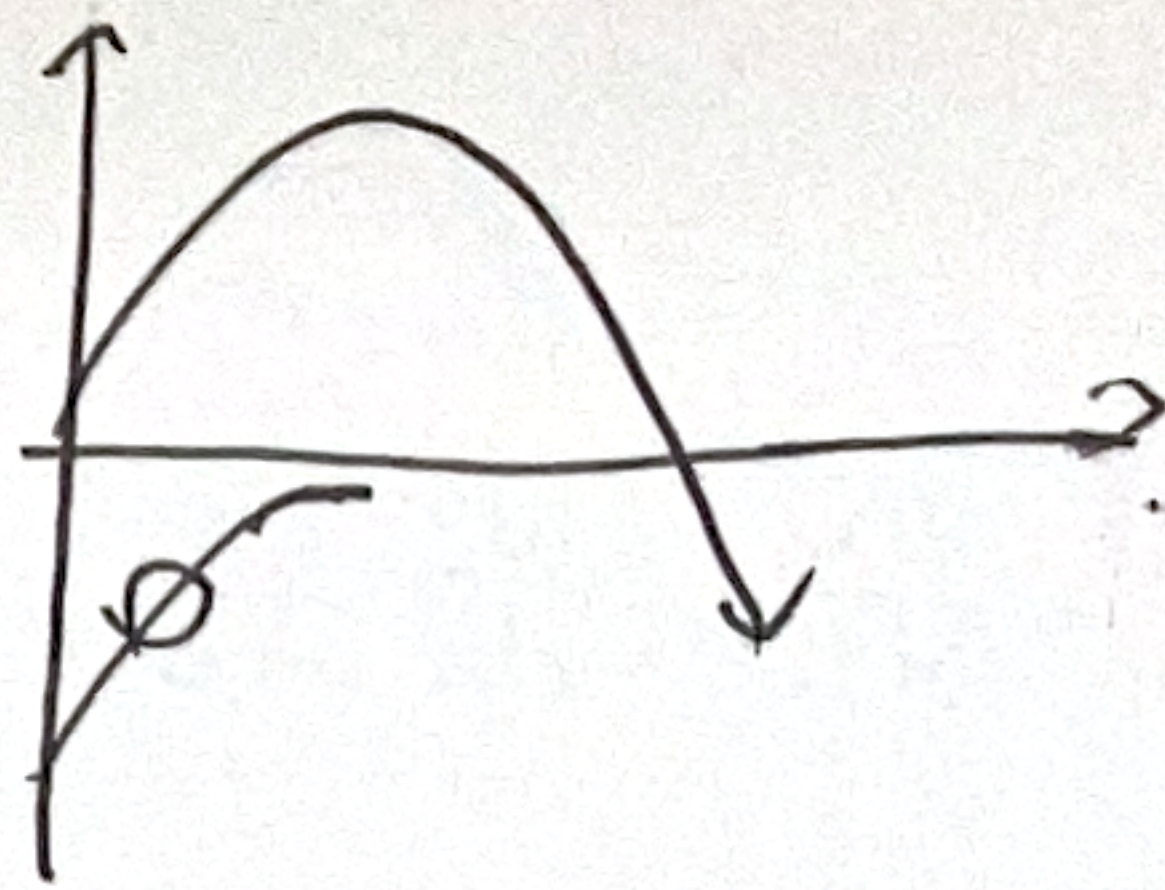
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4.

$f_1 = \frac{1}{2} C_d \rho A v^2 \sim 0.5 \times 0.47 \times 1.225 \text{ kg/m}^3 \times \pi (0.22)^2 \sim 2 \text{ cm} : 3.6 \times 10^{-4} \text{ kg/m}$

$f_{in} f_2 = 3 \pi \eta D v \sim 3 \times \pi \times 1.8 \times 10^{-5} \text{ Pa.s} \sim 2 \text{ cm} : 6.8 \times 10^{-5} \text{ kg/s}$

so ignore  $f_2$

$m \frac{dv}{dt} = mg - cv^2 \Rightarrow \frac{dv}{dt} = g - \frac{c}{m} v^2$

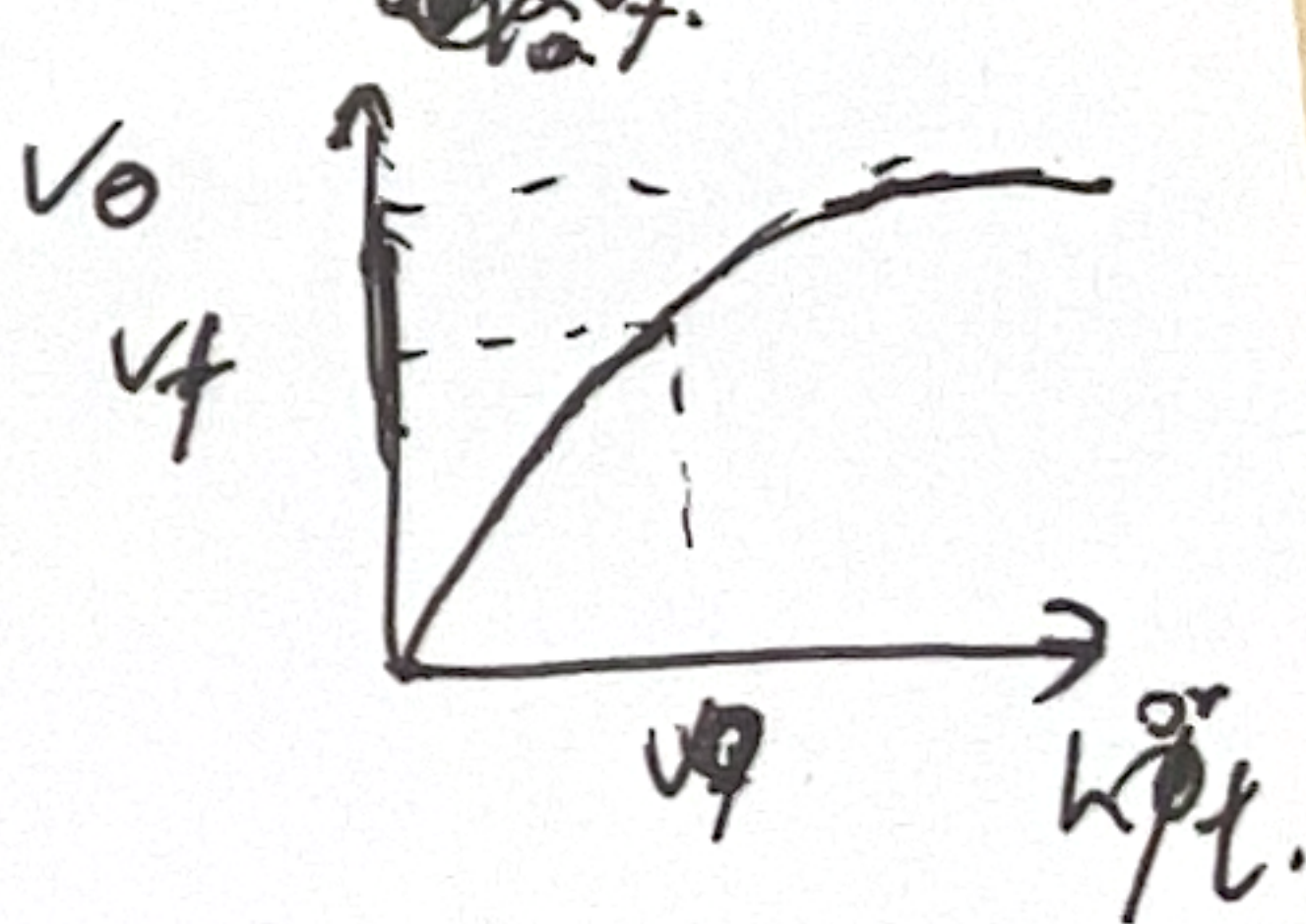
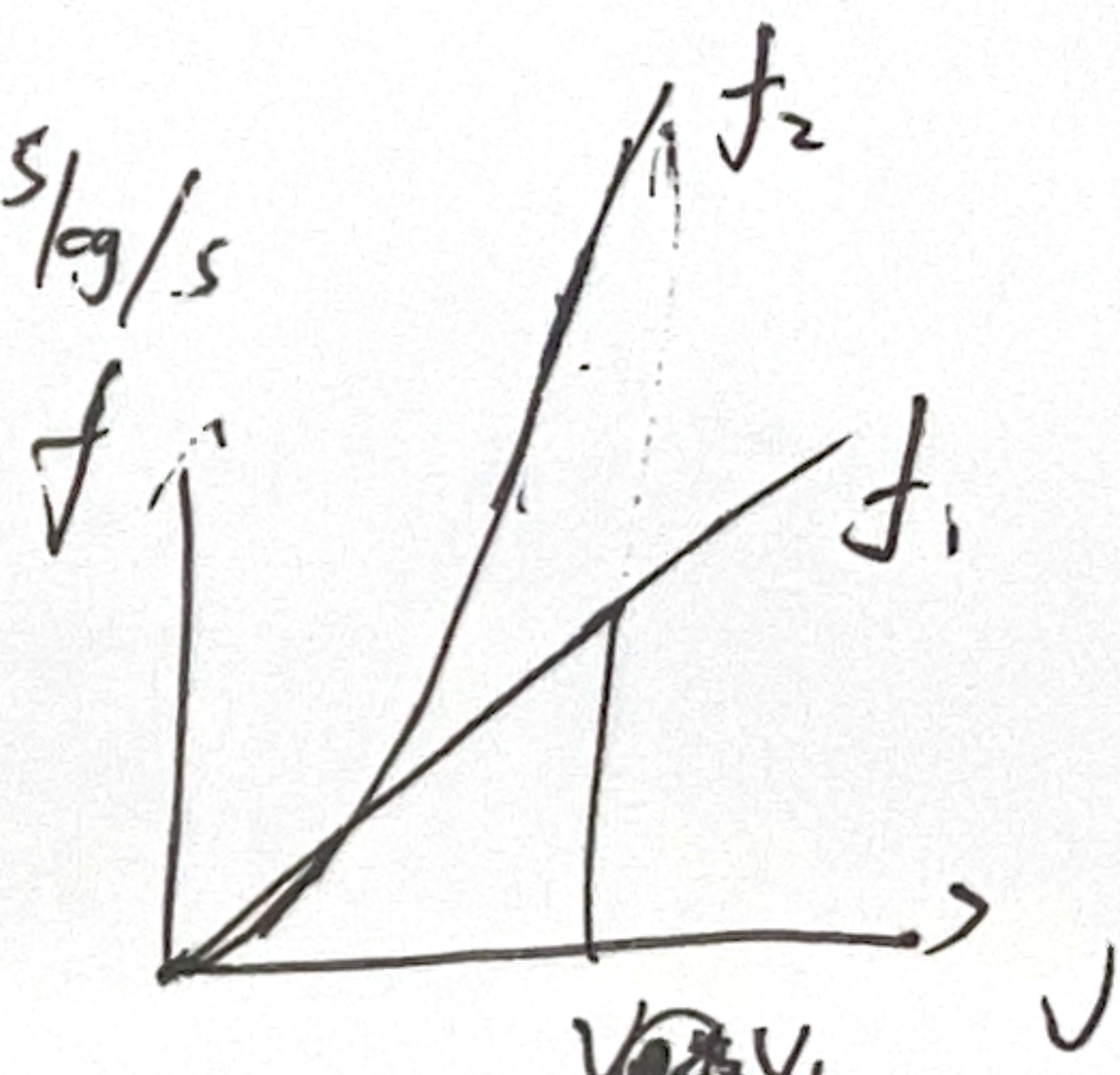
$\int_0^v \frac{dv}{g - (\frac{c}{m}) v^2} = \int_0^t dt$

$\Rightarrow v(t) = \sqrt{\frac{mg}{c}} \tanh\left(\frac{gt}{\sqrt{\frac{mg}{c}}}\right)$

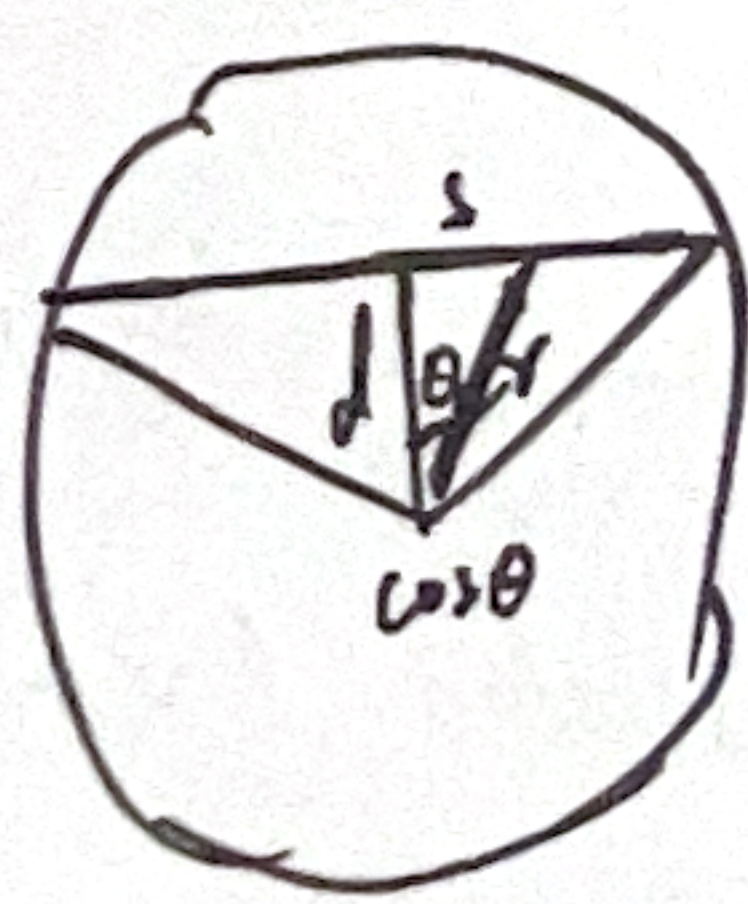
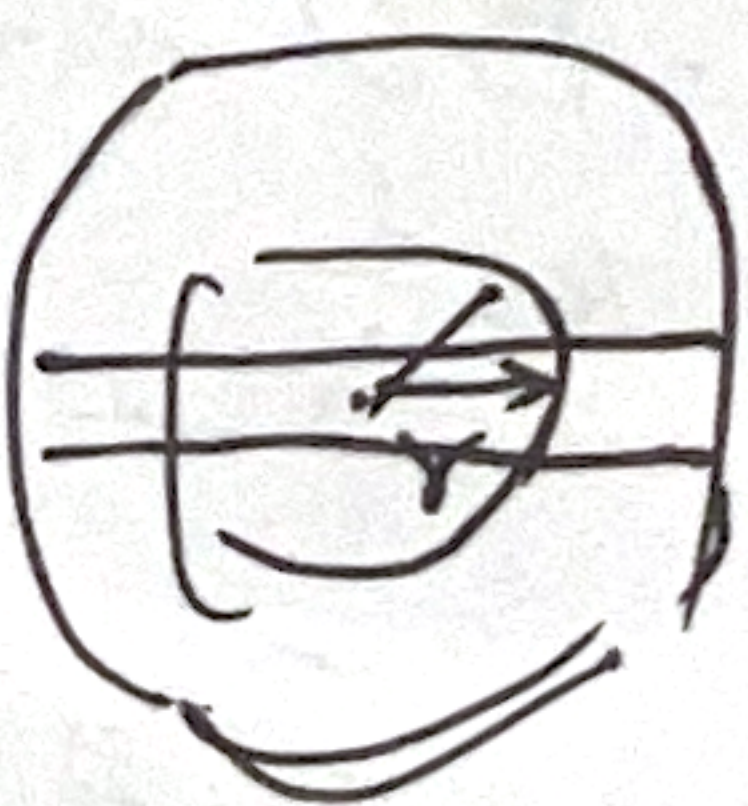
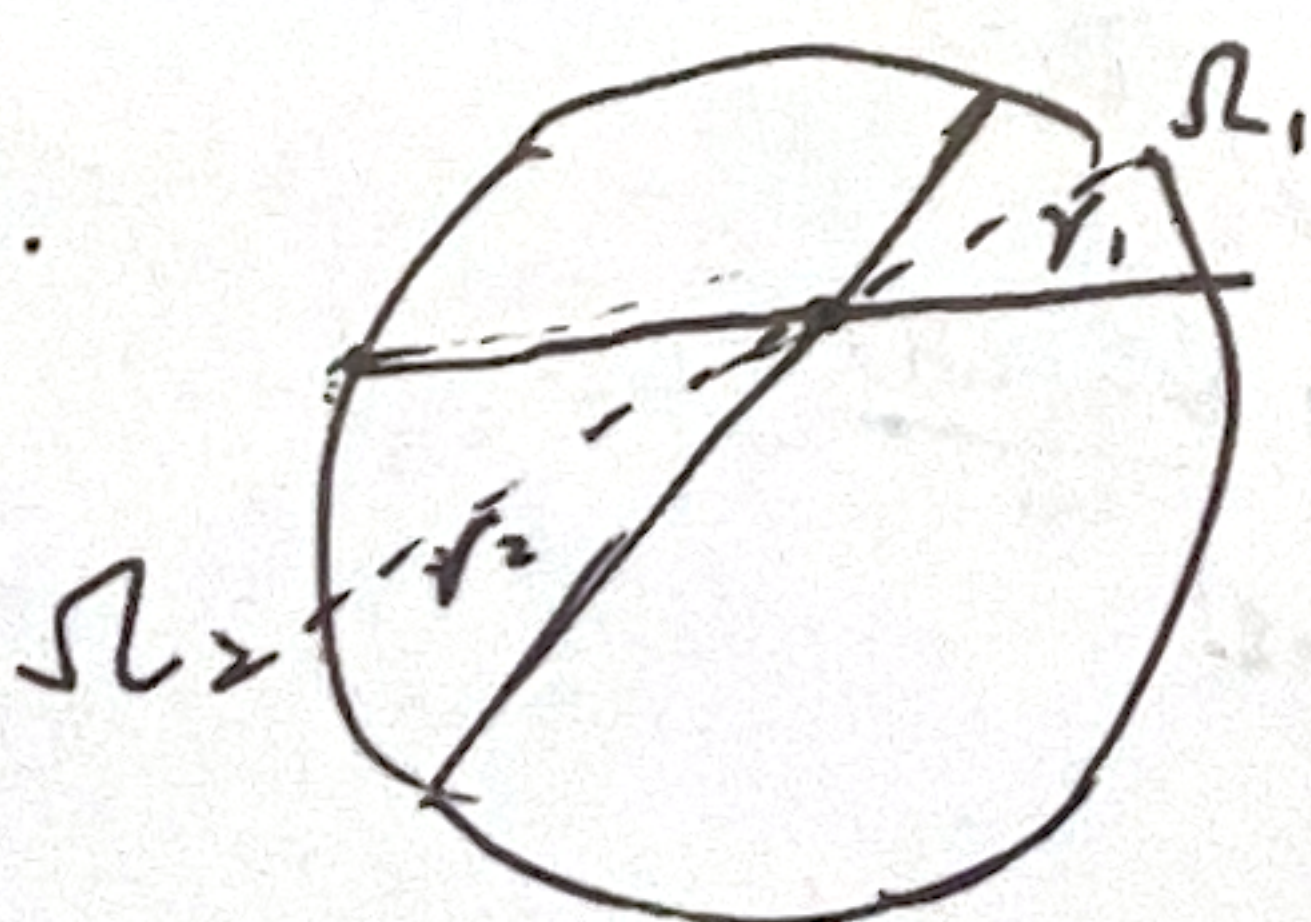
$z(t) = \frac{v_0^2}{g} \ln \cosh\left(\frac{gt}{v_0}\right)$

$z(t_f) = h \Rightarrow t_f$

$\Delta t \sim 0.01 / 0.1 / \sim [0.01, 0.4]$



5.



$F = \frac{GmM}{r^2} = \frac{Gm \rho_s \cdot \frac{4}{3} \pi r^3}{r^2} = Gm \rho_s r$

$M' = \frac{M r^3}{R^3}$

$F(r) = - \frac{GM'm}{r^2} = - \frac{GMm}{R^3} r \Rightarrow \ddot{r} = - \frac{g}{R} r$

$g = \frac{GM}{R^2}$

$\omega = \sqrt{\frac{g}{R}} \quad T = 2\pi \sqrt{\frac{R}{g}}$

$\sim 5000$

$R \sim 6.37 \times 10^6 \text{ m}$   
 $g \sim 9.81 \text{ m/s}^2$

$\ddot{r} = - \frac{g}{R} r$

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$\omega = \sqrt{\frac{g}{R}} \quad T = 2\pi \sqrt{\frac{R}{g}}$

Or  $U(r) = - \frac{GMm}{2R^3} r^2 + \text{constant}$

$= - \frac{1}{2} k r^2$