1 Problem 1:

Skip.

2 Problem 2:

2.1 1

Proof: $((\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T)$, so

$$(\boldsymbol{R}\vec{r_1})^T \cdot (\boldsymbol{R}\vec{r_2}) = \vec{r_1}^T\boldsymbol{R}^T \cdot \boldsymbol{R}\vec{r_2} = \vec{r_1}^T\boldsymbol{I}\vec{r_2} = \vec{r_1}^T\vec{r_2} = \vec{r_1} \cdot \vec{r_2}$$

Or you can say the length and angle of vector not change with the rotation. But you can't use a special example of rotation to prove it.

2.2 2

Proof:

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \hat{i}(b_2c_3 - b_3c_2) - \hat{j}(b_1c_3 - b_3c_1) + \hat{k}(b_1c_2 - b_2c_1)$$

$$\begin{aligned} \vec{a} \cdot (\vec{b} \times \vec{c}) &= a_1(b_2c_3 - b_3c_2) + a_2(b_3c_1 - b_1c_3) + a_3(b_1c_2 - b_2c_1) \\ &= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \end{aligned} \tag{1}$$

Cyclic permutation of rows (or columns) does not change the determinant value.

If you said it equals its volume or expand all items, all right.

Proof: $dW = q(\vec{v} \times \vec{B}) \cdot \vec{v}\,dt = q(\vec{B} \cdot (\vec{v} \times \vec{v}))dt = 0$

2.3 3

Just expand it ...

2.4 4

Use the conclusion fo (3), consider $\vec{U} = \vec{a} \times \vec{b}$, we have $\vec{U} \times (\vec{c} \times \vec{d}) = (\vec{U} \cdot \vec{d})\vec{c} - (\vec{U} \cdot \vec{c})\vec{d}$

$$(\vec{U} \cdot \vec{d})\vec{c} = (\vec{a} \times \vec{b} \cdot \vec{d})\vec{c} \tag{2}$$

Similarly, $(\vec{U} \cdot \vec{c})d = (\vec{a} \times \vec{b} \cdot \vec{c})\vec{d}$

3 Problem 3

Many students misunderstand this question but actually it's a very naive question.

Polar vectors represent "translational symmetry": Their mirror behavior matches intuitive expectations (e.g., a forward-pointing vector in reality points backward in a mirror).

Axial vectors represent "rotational symmetry" and handedness: They arise from cross products, which depend on the right-hand rule. Mirror reflection inverts "handedness" (left right), so axial vectors' mirror behavior is opposite to polar vectors for parallel mirrors, and identical for perpendicular mirrors. This reflects a deeper distinction between "translational" and "rotational" quantities in physics.

4 Problem 4

For the ΔL in L, we have $v = \sqrt{2gL}$, so $\Delta t_i = \frac{\Delta L}{\sqrt{2gL_i}}$.

Now we prove $\sum \Delta t_i$ is a limited value.

Actually,

$$T = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{L/n}{\sqrt{2gL_i}} = \int_0^H \frac{dL}{\sqrt{2gL}} = \sqrt{\frac{2H}{g}} \tag{3}$$

You can't describe it by words or skip the integral and just say it's limited.

5 Problem 5

$\frac{1}{\rho} = \frac{|\frac{d^2y}{dx^2}|}{\left(1+(\frac{dy}{dx})^2\right)^{3/2}}$

5.1 1

Consider $y = \sqrt{R^2 - x^2}$,
so, $\frac{dy}{dx} = -\frac{x}{\sqrt{R^2-x^2}}$,
$\frac{d^2y}{dx^2} = \frac{-R^2}{(R^2-x^2)^{3/2}}$,
and $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{x^2}{R^2-x^2} = \frac{R^2}{R^2-x^2}$.
Hence the conclusion.

5.2 2

At the highest point $v_x = v_0 \cos\theta, v_y = v_0 \sin\theta$. Gravity has **no component** along the horizontal (tangential) direction, $a_\tau = 0$. For the normal acceleration $a_n = v^2/\rho$, we need know the r of the approximate cycle.
$x = v_0 \cos\theta \cdot t \implies t = \frac{x}{v_0 \cos\theta}$, $y = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$,
We can rewrite $y = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$.
Get $\frac{dy}{dx} = \tan\theta - \frac{gx}{v_0^2\cos^2\theta}$, at the highest point $x = v_0\cos\theta \cdot \frac{v_0 \sin\theta}{g}$, so we get $\frac{dy}{dx} = 0$.
$\frac{d^2y}{dx^2} = -\frac{g}{v_0^2\cos^2\theta}$
So $\frac{1}{\rho} = \frac{\left|-\frac{g}{v_0^2\cos^2\theta}\right|}{(1+0^2)^{3/2}} = \frac{g}{v_0^2\cos^2\theta}$
$a_n = v^2/\rho = \frac{g(v_0\cos)^2}{v_0^2\cos^2\theta} = g$, hence the conclusion.