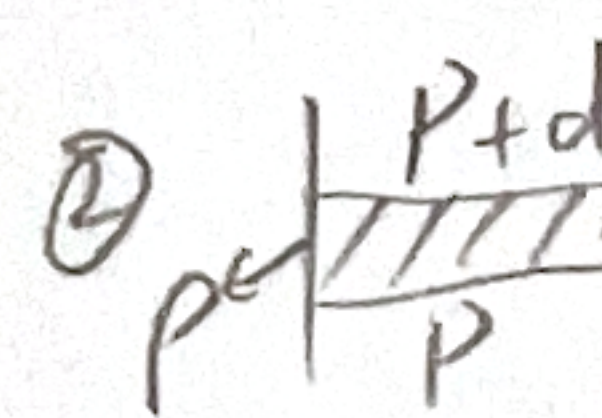


$$1. \quad \frac{dT}{dz} = \frac{\frac{dT}{dP}}{\frac{dP}{dz}} = \frac{T}{1 \cdot P_{vap}} \cdot \frac{P_0 M_{air}}{R T_a} (-g) = \frac{373}{2.44 \times 10^6 \times 0.598} \times \frac{1.01 \times 10^5 \times 0.029}{8.31 \times 300} \times (-9.8) \approx 3 \times 10^{-3} \text{ K/m}$$

①: Clausius-clapeyron equation: $\frac{\partial P}{\partial T} = \frac{1}{T \Delta V} \approx \frac{1}{T V_{vap}} = \frac{1}{T} \frac{P_{vap}}{T_a}$

②  $P + dP = P - P_0 g dz \Rightarrow \frac{dP}{dz} = -P_0 g = -\frac{P_0 M_{air}}{R T_a} \cdot g$

2. (a) linear relation: $\frac{dT}{dz} = -\frac{\Delta T}{z} \Rightarrow \text{heat flow: } \frac{dq}{dt} = -k \frac{dT}{dz} = k \frac{\Delta T}{z}$
for unit area latent heat: $dq' = L \cdot P_{ice} dz$

$$dq = dq' \Rightarrow k \frac{\Delta T}{z} dt = L P_{ice} dz$$

$$\frac{k \cdot \Delta T}{L P_{ice}} dt = z dz \Rightarrow \frac{k \cdot \Delta T}{L P_{ice} z} = \frac{dz}{dt}$$

↓

(b)

$$\int_0^t \frac{k \cdot \Delta T}{L P_{ice}} dt = \int_0^z z dz$$

$$\Rightarrow \frac{k \cdot \Delta T \cdot t}{L P_{ice}} = \frac{1}{2} z^2$$

$$z = \sqrt{\frac{2k \cdot \Delta T \cdot t}{L P_{ice}}} \approx \sqrt{\frac{2 \times 2 \times 10 \times 3600}{900 \times 33 \times 10^5}} \approx 2.2 \text{ cm.}$$

3. for $dA = Bdc + Vde \Rightarrow B = \left(\frac{\partial A}{\partial c}\right)_e, V = \left(\frac{\partial A}{\partial e}\right)_c \Rightarrow \frac{\partial^2 A}{\partial c \partial e} = \frac{\partial^2 A}{\partial e \partial c} \Rightarrow \frac{\partial B}{\partial e} = \frac{\partial V}{\partial c}$

for U, $F = U - TS, H = U + PV, G = U - TS + PV$

(a) $dU = Tds - PdV \Rightarrow \left(\frac{\partial T}{\partial V}\right)_s = -\left(\frac{\partial P}{\partial S}\right)_V$

$dF = -SdT - PdV \Rightarrow \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

$dH = Tds + VdP \Rightarrow \left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial V}{\partial S}\right)_P$

$dG = -SdT + VdP \Rightarrow \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

(b): $ds = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$

$$= \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T \left[\left(\frac{\partial V}{\partial P}\right)_T dP + \left(\frac{\partial V}{\partial T}\right)_P dT\right]$$

$$= \left[\left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P\right] dT$$

$$+ \left[\left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T\right] dP$$

$$ds = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

$$\Rightarrow \left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$$

(c) $dU = Tds - PdV$

$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial S}{\partial V}\right)_T - P \oplus \left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$ [in (a)]

$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P$

(d) $dH = Tds + VdP$

$\left(\frac{\partial H}{\partial P}\right)_T = T \left(\frac{\partial S}{\partial P}\right)_T + V \oplus \left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$ [in (a)]

$\left(\frac{\partial H}{\partial P}\right)_T = -T \left(\frac{\partial V}{\partial T}\right)_P + V$

$$4. 1.) W = P(V_g - V_e) - (P - dP)(V_g - V_e) \approx V dP$$

$$\eta = \frac{W}{Q_h} = \frac{dT}{T}$$

$$\frac{W}{L} \approx \frac{V dP}{L}$$

$$\Rightarrow \frac{dP}{dT} = \frac{L}{VT}$$

$$(b) \frac{V}{L} dP = \frac{dT}{T} \quad \oplus \quad PV = RT \Rightarrow V = \frac{RT}{P} \quad \text{or} \quad V = \frac{Nk_B T}{P}$$

$$\Rightarrow \frac{dP}{P} = \frac{L dT}{RT^2}$$

$$\int \frac{dP}{P} = \int \frac{L dT}{RT^2}$$

$$\ln P = -\frac{L}{R} \frac{1}{T} + C$$

$$P = P_0 \exp\left(-\frac{L}{RT}\right) \quad \text{or} \quad P = P_0 \exp\left(-\frac{L}{Nk_B T}\right)$$

$$(c) \quad T_1 = 25^\circ \text{C} \approx 298 \text{K} \quad T_2 \approx -18.5^\circ \text{C} \approx 254.5 \text{K}$$

$$\eta_{\max} = 1 - \frac{T_2}{T_1} \approx 14.6\%$$

$$Q = m \cdot L = 2.07 \times 10^{17} \text{ J}$$

$$P_{\max} = \eta_{\max} \cdot \frac{Q}{\Delta t} = 14.6\% \times \frac{2.07 \times 10^{17} \text{ J}}{3600 \text{ s}} \approx 8.4 \times 10^{12} \text{ W}$$

