Problem 1: Kinetic Energy Machine
In class, we have talked about the "kinetic energy machine", and classified them as reversible ones and irreversible ones.

Consider an object rolls down lowering its height at h from a kinetic energy machine. Based on the impossibility of perpetual motion machine, please prove that the speed is the highest from a reversible machine. And for two reversible machines, the speeds are the same.

Problem 2: Total Energy of a Many-particle System
Read pages 5-7 in the lecture note 9, and do the following problem.

Consider a cluster of N point particles in an external uniform gravity field. There exists interaction between any pair of particles i and j. The interaction only depends on their distance, and obeys Newton's 3rd law. Please write down the form of the conserved energy of such a system.

Problem 3: Stability of a cube balanced on a cylinder

Put a cube with the edge length $2b$ on a cylinder fixed on the ground as shown in the figure. The radius of the cylinder is r. Please analyze under what condition, the cube is stable on the cylinder?

Before you do the concrete calculation, you may use your intuition to guess your result and check if your intuition is correct.

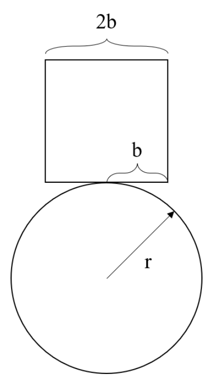

Figure 1: The stability of a cube on a fixed cylinder.

Problem 4: The electric potential of an electric dipole.

For a small charged object, if we measure its electric field at a long distance, it looks just like a point (with a net charge of this object) since its size becomes unimportant. But what if the net charge is zero and the charge of the object is not evenly distributed? In the following, you should discuss a simple example.

This system shown in figure is the so-called the electric dipole, which consists of a positive charge e and a negative charge $-e$ with a relative distance r. Take the limit that $r \to 0$ and $p = er$ remain a constant. Please figure out the electric potential distribution $U(\mathbf{r})$.

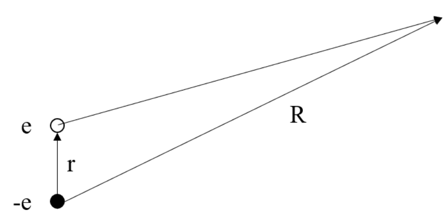

Figure 2: Electric dipole

Problem 5: The tide potential

We have learned that the shape of the ocean is distorted to be an ellipsoid due to the tidal force from the Moon. Based on the tidal force formula,

$$\mathbf{F}_{tide}(\mathbf{r}) = -GMm\left(\frac{\mathbf{d}_0+\mathbf{r}}{|\mathbf{d}_0+\mathbf{r}|^3} - \frac{\mathbf{d_0}}{d_0^3}\right), \tag{1}$$

where $\mathbf{d} = \mathbf{d}_0 + \mathbf{r}$. $\mathbf{d_0}$ is the vector from the moon center to the earth center, and $\mathbf{r}$ is the vector from the earth center to a location on the ocean surface. 1) Please show that the tidal force can be expressed in terms of a potential

$$\mathbf{F}_{tide}(\mathbf{r}) = -\nabla U_{tide}(\mathbf{r}). \tag{2}$$

2) Combine U_{tide} and the earth gravity potential U_{earth} together as the total potential energy. Calculate the height difference of the ocean surfaces between the ebb tide and flood tide $\Delta h = h_{flood} - h_{ebb}$.