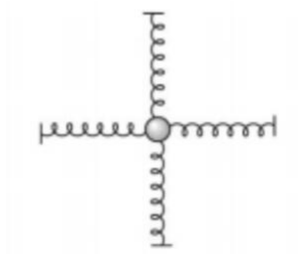
Problem 1: Four springs harmonic oscillation

Consider the mass attached to four identical springs, as shown in Figure. Each spring has force constant k and unstretched length l_0, and the length of each spring when the mass is at its equilibrium at the origin is a (not necessarily the same as l_0). When the mass is displaced a small distance to the point (x, y), show that its potential energy has the form $\frac{1}{2}k'r^2$ appropriate to an isotropic harmonic oscillator. What is the constant k' in terms of k? Give an expression for the corresponding force.



Problem 2: Driven harmonic oscillation

A spar buoy of uniform cross-section floats in a vertical position with a length L submerged when there are no waves on the ocean. Please describe the motion of the spar buoy when there are sinusoidal waves of height h (crest to trough) and the period T on the ocean.

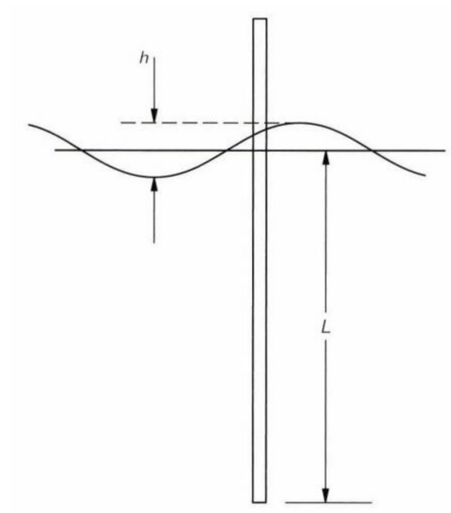


Figure 2: The spar buoy in the ocean

Problem 3: Damped harmonic oscillation I

A mass m is suspended from a spring of force constant k in a medium which exerts a damping form $-m\gamma dx/dt$.

1. For the case of under damped motion find the complete solutions for the position $x = x(t)$ of m for all times $t > 0$ for the following driving forces:

   (a)
$$F = \begin{cases} 0 & \text{for } t < 0 \\ F_0 & \text{for } t \geq 0 \end{cases} \tag{1}$$

   (b)
$$F = \begin{cases} 0 & \text{for } t < 0 \\ F_0 \cos \omega_0 t & \text{for } t \geq 0 \end{cases} \tag{2}$$

$$\omega_0 = \sqrt{k/m}$$

2. If the oscillator is driven by a sinusoidal force $F = F_0 \cos \omega t$ and we consider long times, what is the frequency ω^* for which the amplitude reaches a maximum?

Problem 4: Damped oscillation II

Consider a damped oscillator, with natural frequency ω_0 and damping constant β both fixed, that is driven by a force $F(t) = F_0 \cos \omega t$.

1. Find the rate $P(t)$ at which $F(t)$ does work and show that the average rate $\langle P \rangle$ over any number of complete cycles is $m\beta\omega^2 A^2$.
2. Verify that this is the same as the average rate at which energy is lost to the resistive force.
3. Show that as w is varied $\langle P \rangle$ is maximum when $\omega = \omega_0$; that is, the resonance of the power occurs at $\omega = \omega_0$ (exactly).