Problem 1:

Some one-dimensional kinematics problems can always be reduced to doing an integral. Here is a class of problem: Show that if the net force on a one-dimensional particle depends only on position, $F = F(x)$, then Newton's second law can be solved to find v as a function of x given by

$$v^2 = v_0^2 + \frac{2}{m} \int_{x_0}^{x} F(x')dx'$$

[Hint: Use the chain rule to prove the following handy relation, which we call the "vdv/dx rule": If you regard v as a function of x, then

$$\dot{v} = v\frac{dv}{dx} = \frac{1}{2}\frac{d(v^2)}{dx}$$

Use this to rewrite Newton's second law in the separated form $md(v^2) = 2F(x)dx$ and then integrate from x_0 to x.] Comment on your result for the case that $F(x)$ is actually a constant. (You may recognize your solution as a statement about kinetic energy and work, both of which we shall be discussing in later courses.)

Problem 2:

A charged particle of mass m and positive charge q moves in uniform electric and magnetic fields, $\mathbf{E}$ pointing in the y direction and $\mathbf{B}$ in the z direction (an arrangement called 'crossed E and B fields'). Suppose the particle is initially at the origin and is given a kick at time $t = 0$ along the x axis with $v_x = v_{x0}$ (positive or negative).

1. Write down the equation of motion for the particle and resolve it into its three components. Show that the motion remains in $z = 0$ plane.
2. Prove that there is a unique value of v_{x0} called the drift speed v_{dr}, for which the particle moves undeflected through the field. (This is the basis of velocity selectors, which select particles traveling at one chosen speed from a beam with different speeds.)
3. Solve the equations of motion to give the particle's velocity as a function of t, for arbitrary values of v_{x0}. [Hint: Try to relate the equations with harmonic oscillator.]
4. Integrate the velocity to find the position as a function of t and sketch the trajectory for various values of v_{x0}.

Problem 3:
A gun can fire shells in any direction with the same speed v_0(ignoring air resistance). Let's take cylindrical polar coordinate with the gun at the origin and z measured vertically up. Show that the gun can hit any object inside the surface

$$z = \frac{v_0^2}{2g} - \frac{g}{2v_0^2}\rho^2$$

Problem 4: Fall with resistance

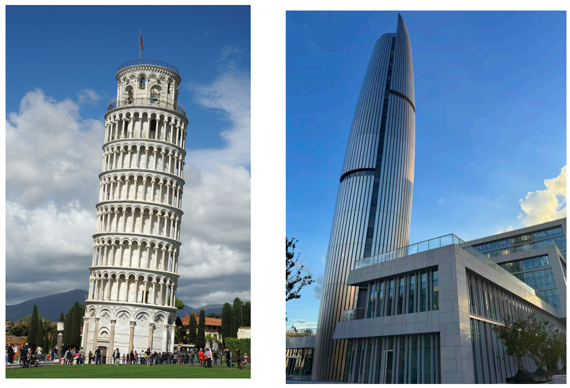


Figure 1: Left: The leaning tower of Pisa. Legend says that Galileo did the freefall experiment to falsify Aristotle's theory. right: the watch tower at Westlake University.

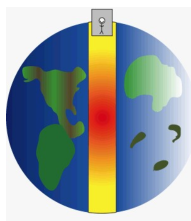
1. Legend has it that Galileo did the free-fall experiments on the leaning tower of Pisa. He dropped two iron balls with different weights and they arrived on the ground simultaneously. However, due to the air resistances, rigorously speaking, they should fall differently.

   Suppose that the radius of the bigger iron ball is 2cm, and that of the smaller one is 1cm. The leaning tower of Pisa is 55m. Using the knowledge and parameter values provided in the lecture note, calculate the difference between the falling times of these two iron balls. Would Galileo be able to tell the difference?

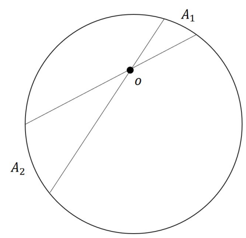
   2. Westlake Watch Tower(temperary name)is roughly 108 meters high. If someone throw an watermelon from the top of the tower (do not to it in real life!). The radius of watermelon is about 20cm. Now try to evaluate the speed of watermelon when it hits the ground by taking into account the air resistance.

Problem 5: The Longest Fall

In Liu Cixing's scientific fiction "The earth cannon (the longest fall)", a new method to travel between any two points on the earth is described. Consider two endpoints of a diameter of the earth, which are farthest from each other. You build a tunnel running through the interior of the earth, and just jump into it and fall to the other end as shown in Fig 1 (a). You do not need to worry about the internal structure of the earth and assume that the density of the earth is uniform.



(a) Gravity elevator



(b) Spherical shell

Figure 2: The longest fall

1. We can firstly study the gravity inside a spherical shell. Please make the argument that, there is no gravity inside the spherical shell.
   (Hint: Consider the situation in Figure (b). For an arbitrary point inside the spherical shell, verify that the forces generated by A_1 and A_2 cancel each other.)

2. For a point outside the spherical shell, you can treat the gravity as if all of its mass are concentrated in the center of the shell. This is a highly non-trivial statement that Newton spent many years to prove it. You do not need to prove it here, and just take it as a fact.

   Then please prove that you will do a harmonic oscillation after you jump into the tunnel. What is the period of the motion?

3. Consider a man-made satellite skimming the surface of earth. Please verify that your motion is just the projection of the circular motion of the satellite to the tunnel.

4. To be more practical, we would like to build the tunnel (still straight) to connect two general points on the earth, i.e., the tunnel does not need to pass the earth center.

In this case, you may sit in an elevator in this tunnel. The elevator has no power and its motion is confined along the tunnel. Please describe the motion of this gravity elevator and calculate its period. (For simplicity, you can regard the motion is frictionless)