HW11: Due on Dec 18 in class. CODE NUMBER: ————— SCORE: ————

Problem #1: Boiling water on a mountain

The latent heat of vaporization of water is about 2.44×10^6 J kg^{-1}, and the vapor density at 100 °C is 0.598 kg m^{-3}. Use the Clausius-Clapeyron Equation to find the rate of change of the boiling temperature with altitude, dT/dz, near sea level in K km^{-1}. Assume the temperature of the air is 300 K.

Problem #2: A freezing lake

The top layers of the water in a lake are initially at 0°C. A cold breeze starts to blow, keeping the surface of the lake at a temperature ΔT below 0°C.

(a) Find the rate of increase of the thickness of the ice dz/dt, assuming that the heat loss required to lower the temperature of the ice which has already formed is much less than that required to form new ice. (Assume also that the temperature in the ice changes linearly with depth)

(b) Using the result of part (a), calculate the thickness of the ice z one hour after freezing begins if ΔT = 10 °C.

Use: thermal conductivity of ice: κ= 2.0 W m^{-1} K^{-1}, latent heat of ice formation: L = 3.3 $\times 10^5$ J kg^{-1}.

Problem #3: Thermal potentials and their derivatives

In class, we have learnt a variation of thermodynamic potential U, F, H and G. Using their definition, and considering their partial derivatives of S, p, T and V, show the derivation of:

(a) the four different Maxwell relations

(b) $\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial S}{\partial T}\right)_V + \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P$

(c) $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$

(d) $\left(\frac{\partial H}{\partial P}\right)_T = -T\left(\frac{\partial V}{\partial T}\right)_P + V$ (do not use Feynman's trick P→−V, V→P)

Problem #4: Clausius–Clapeyron equation

Clausius–Clapeyron equation describes the variation of boiling point with pressure. It is usually derived from the condition that the chemical potentials of the gas and liquid phases are the same at coexistence. For an alternative derivation, consider a Carnot engine using one mole of water. At the source (P, T) the latent heat L is supplied converting water to steam. There is a volume increase V associated with this process. The pressure is adiabatically decreased to P − dP. At the sink (P − dP, T − dT) steam is condensed back to water.

(a) Show that the work output of the engine is $W = V\,dP + O(dP^2)$. Hence obtain the Clausius–Clapeyron equation

$$\frac{dP}{dT} = \frac{L}{TV}. \qquad (1)$$

(b) Assume that L is approximately temperature independent, and that the volume change is dominated by the volume of steam treated as an ideal gas, i.e., $V = N\,k_B T/P$. Integrate equation (1) to obtain P(T).

(c) A hurricane can be compared to an engine. It begins with the evaporation of water from the warm ocean surface, where the temperature is around 25°C. This water vapor rises into the atmosphere and eventually condenses into water at higher, cooler altitudes with temperatures around -185°C. The Coriolis effect transforms this upward movement of air into a spiraling motion, similar to creating a miniature storm in a teacup using ice and boiling water. For this process to be effective, the warm surface layer of the ocean needs to be at least 60 meter deep to supply enough water vapor. A hurricane must condense about 90 million tons of water vapor every hour to sustain itself. Given these conditions and the latent heat of vaporization of water being approximately 2.3×10^6 joules per kilogram. Calculate the maximum possible efficiency, and power output of such a hurricane.