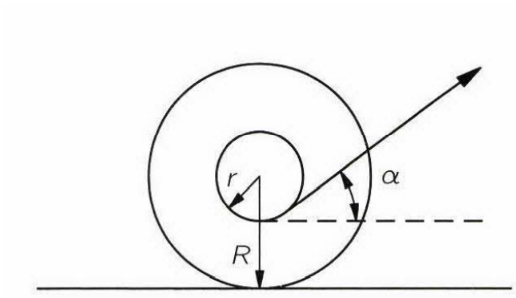
Problem 1: Angular momentum conservation

An air puck of mass m moves on the surface of a horizontal table, guided by a string attached to the puck and passing downward through a small hole in the table top. Initially the length of string above the table is r_1, and the puck is set moving at speed v_1 in a circular path of this radius. The string is then pulled downward through the hole until an amount r_2 remains above the table. Find:

1. The final speed v_2 of the puck.
2. The work W required to pull the string through the hole from r_1 to r_2.
3. The magnitude of the force F needed to hold the radius at a constant r.

Problem 2 A toy of yo-yo

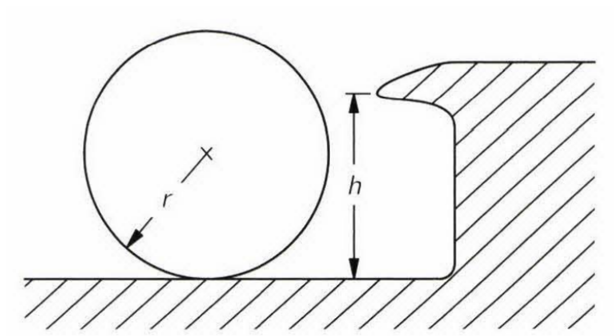
A yo-yo of mass M, outer radius R, and moment of inertia I, on a horizontal table is free to roll without slipping. A force F is applied at the inner radius r at an angle α with the horizontal, as shown in figure.



1. Find the acceleration a of the yo-yo, if the yo-yo does not rise from the table top.
2. How strong a force F at the angle α is needed in order to lift the yo-yo off the table?

Problem 3 Bounce back

Find the ratio h/r of the height of the cushion of a billiard table to the radius of the balls, as shown in figure, such that a ball that approaches the cushion with a pure rolling motion will rebound with a pure rolling motion even if the coefficient of friction between the ball and the table is negligible. Assume that the force exerted on the ball by the cushion during the impact is in the horizontal direction.



Problem 4: Euler equations

A rigid body is rotating freely, subject to zero torque.
1) Use the Euler equation to prove that the magnitude of the angular momentum **L** is a constant.

2) Prove that the kinetic energy of rotation $T = \frac{1}{2}(\lambda_1 \omega_1^2 + \lambda_2 \omega_2^2 + \lambda_3 \omega_3^2)$ is a constant.