Homework 7

Due: November 10, 2026 Code number: _____ Score: _____

Problem 1: Pulling a puck toward a hole

An air puck of mass m moves without friction on a horizontal table. A massless, inextensible string is attached to the puck and passes through a small, frictionless hole in the table. Initially, the puck moves in a circle of radius r_1 with speed v_1.

The string is pulled downward until the length above the table is r_2, with $0 < r_2 < r_1$. At the end of the pull, the radial velocity is zero and the string is held at this length, so the puck again moves in a circle. The tension is the only horizontal force on the puck.

1. Find the final speed v_2 of the puck.

2. Find the work W done on the puck by pulling the string from r_1 to r_2.

3. At an intermediate stage, the string is held at a fixed length r. Find the magnitude of the force needed to maintain circular motion at that radius, in terms of m, r_1, v_1, and r.

Problem 2: Pulling a spool

A spool of mass M has outer radius R, inner hub radius r, and moment of inertia I about its symmetry axis through the center of mass, with $0 < r < R$. It rests on a horizontal table. A light string is wound around the inner hub and pulled with force F at an angle α above the horizontal, where $0 < \alpha < \pi/2$.

The string leaves the lower side of the inner hub, as shown in Fig. 1. The applied force therefore produces a counterclockwise torque of magnitude Fr about the center. Take horizontal acceleration to the right as positive.

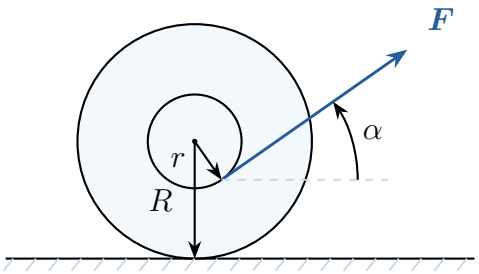


Fig. 1: The string is tangent to the inner hub. The outer rim contacts the table.

1. Assume that the spool remains in contact with the table and rolls without slipping. Find its horizontal acceleration a.

2. Allow the spool to slip if necessary. From the vertical force balance while it is in contact with the table, find the force F at which the normal reaction reaches zero. The no-slip assumption in part 1 is not imposed in this part.

Problem 3: A billiard ball rebounding from a cushion

A uniform solid sphere of mass m and radius r approaches the cushion of a billiard table in pure rolling motion. Its moment of inertia about a diameter is

$$I = \frac{2}{5}mr^2.$$

During the short collision, the cushion exerts a horizontal impulse on the ball at height h above the table. Neglect any tangential impulse from the table. The ball rebounds horizontally, and its center remains at height r during the collision. The cushion's impulse is the only horizontal impulse on the ball.

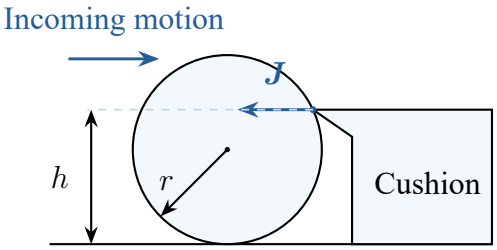


Fig. 2: The cushion impulse $\boldsymbol{J}$ acts horizontally at height h. The diagram is not to scale.

Find the ratio h/r for which the ball is also in pure rolling motion immediately after rebounding. Use both linear and angular impulse–momentum relations. A coefficient of restitution is not needed.

Problem 4: Conserved quantities in torque-free rotation

A rigid body rotates freely with zero external torque about its center of mass. Let $\lambda_1, \lambda_2, \lambda_3 > 0$ be its principal moments of inertia, and let $\omega_1, \omega_2, \omega_3$ be the angular-velocity components along the corresponding body-fixed principal axes. Dots denote time derivatives of these components.

You may use the torque-free Euler equations:

$$\lambda_1\dot{\omega}_1 = (\lambda_2 - \lambda_3)\omega_2\omega_3,$$
$$\lambda_2\dot{\omega}_2 = (\lambda_3 - \lambda_1)\omega_3\omega_1,$$
$$\lambda_3\dot{\omega}_3 = (\lambda_1 - \lambda_2)\omega_1\omega_2.$$

1. Use these equations to prove that the angular-momentum magnitude is constant. The body-frame components are $L_i = \lambda_i\omega_i$; consider the time derivative of L^2.

2. Prove that the rotational kinetic energy

$$T = \frac{1}{2}\left(\lambda_1\omega_1^2 + \lambda_2\omega_2^2 + \lambda_3\omega_3^2\right)$$

is constant.