Problem 1: Momentum conservation from Galilean relativity principle

In class, we have learned the Galilean relativity principle and Galilean transformation. Below we will see it bridges two important conservation laws, energy and momentum conservations.

Consider that two masses m_1 and m_2 with initial velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, respectively, collide, and the final velocities become $\mathbf{v_1}'$ and $\mathbf{v_2}'$, respectively.

1) If this process is elastic, i.e., the kinetic energy is conserved, then no energy should be converted into heat. This fact should remain to be true in any inertial reference frame according to the relativity principle. Based on this fact, prove that total momentum is conserved during this collision process.

2) If this process is *inelastic*, i.e. there is a certain amount of kinetic energy loss converted into heat. Should the total momentum remain conserved or not in the inelastic collision process? Please give your reasoning based on Galilean relativity principle and Galilean transformation.

Problem 2: The Roche limit – Will the earth be torn apart by the Jupiter or not?

In the science fiction movie (not in the novel!) *"The Wandering Earth"*, the earth would be falling into Jupiter and facing the risk of disintegration in the beginning of its lonely journey of 2500 years. Indeed, when two celestial objects are too close to each other, one, or, both of them could be torn apart by the tidal force. The critical distance is called the Roche limit.

But you will see that the movie director should take our class in advance to avoid a mistake. Anyway, since this plot is not in Liu Cixin's book, I do not criticize Liu here. In contrast, the executive producer of "*Interstellar*" **is** a physics professor at Caltech.

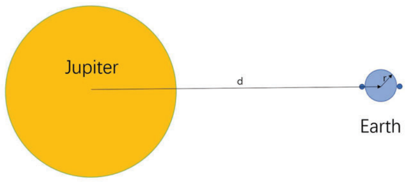

Figure 1: Roche limit of Earth and Jupiter

1. Consider the situation of bringing the earth and the moon together with a distance d between their centers. Look at a rock on the moon located on the side facing the earth, which stays on the moon only due to the moon's gravity.

   The earth radius is denoted as R. Please convince yourself that if d/R is smaller than a critical value, then the rock will fly away from the moon. Of course, d needs to be larger than the earth radius R to be meaningful. Does your result satisfy this requirement or not? (Hint, the moon's average density is $3.3g/cm^3$ and its radius is 1737km. The earth's average density is 5.5 g/cm^3 and its radius is 6378km.)

   Please use the rock on the back side of the moon to do this analysis. Convince yourself that you arrive at the same result at the leading order of approximation.

2. Now come back to the case of the movie that the earth is falling into the Jupiter. Please decide whether the earth will be disintegrated or not before crashing into Jupiter? (Hint: the average density of Jupiter is $1.3g/cm^3$ and its radius is $71493km$.)

Problem 3: The tidal locking

Tidal locking is a phenomenon commonly observed in a pair of co-orbiting celestial objects like the earth and moon. The moon is already locked since its spin period and its revolution period around the earth are the same. But the earth spins much faster, hence, it has not been locked so far. In this problem, we are going to study the reason and effect of tidal locking.

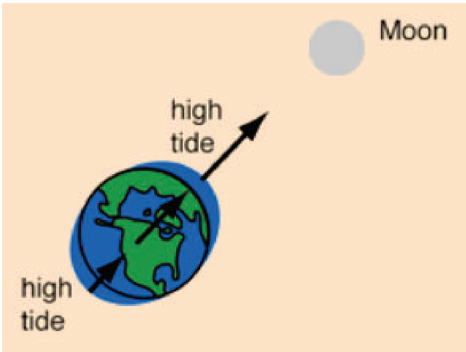

Figure 2: The ocean surface will be deformed to an ellipsoid shape under the gravity of the moon.

1. Assume that the ocean covers all of the earth surface. The shape of the ocean surface can be easily deformed by gravity of the moon, as illustrated in Figure 2. Explain why it exhibits an ellipsoid shape. This explains why there are two ebb (low) and flood (high) tides a day.

2. Naively, we would expect that the bulge of ocean should be along the line connecting the earth-moon centers. However, the earth spins is at a much faster angular speed than the moon's revolution. Hence, friction exists between the ocean and the seebed, which slows down the earth spin rate.

   Then the earth loses its angular momentum, where can this angular momentum go to?

   Based on this knowledge, explain why the moon will be away from us, and how could the earth be tidally locked if we wait for a sufficiently long time?

3. Observation tells us that the distance between moon and earth increases 3.8cm every year. Based on angular momentum conservation, try to evaluate how many hours a day during the geologic period of Devonian (370 million years ago), in which that dinosaurs lived?

   [Hint: The angular momentum of a rotating ball is given by $J_M = \frac{2}{5}MR^2\omega$, where M is the mass of ball, R is the radius, and ω is the angular velocity. You can dismiss the angular momentum from the moon's spin, since it is way smaller than that of earth's spin and moon's revolution.]