Problem 1: The hyperbolic orbit

In class, we have learned that the general orbit of a planet's motion is an ellipse. Nevertheless, there is another possibility that the orbit could be one branch of a hyperbola. In this case, the orbit is not closed. This kind of orbits apply for certain comets which just come once and never return. Please use the similar geometric method that you learned in class, prove the following statements without calculus.

1. Kepler's 2nd law still applies, i.e., the areas swept by the line connecting the Sun and the comet

2. Prove that another possibility of orbit in the solar system is one branch of a hyperbola. Under what condition does it happen?

   Hint: Consider its trajectory in the velocity space. Check whether it encloses the origin of the velocity space.

Problem 2: A parabolic orbit

Between ellipses and hyperbolas, another type of conic curves are parabolas. Under what condition is a comet's orbit a parabola by checking its trajectory in the velocity space.

Problem 3: Repulsive inverse-square force field

In Rutherford's experiment, an α particle is scattered by an atomic nucleus. Since both of them carry positive charges, the force between them is a repulsive inverse square force.

1. Prove that the Kepler's 2nd law is still valid.
2. Prove that the general orbit is one branch of a hyperbola by the geometric method.

Problem 4: Proof of Kepler's First Law

In class, we showed a proof of Kepler's First Law starting from the inverse-squared Newtonian Gravity. Derive the complete proof using calculus. (you can start from the Lecture Slide "Proof of the first law using calculus", make sure you show your derivation step-by-step)