

Homework 4

Due: October 20, 2026

Code number: _____

Score: _____

Problem 1: Open orbits in an attractive force field

A comet moves under the gravitational attraction of the Sun,

$$\mathbf{F} = -\frac{GMm}{r^2} \hat{\mathbf{r}}.$$

Treat the Sun as fixed and assume nonzero angular momentum. In Lecture 6, the endpoints of the velocity vectors, drawn from a common origin O on fixed Cartesian axes, were shown to lie on a circle. Let C be its center, V_c its radius, and $w = |OC|$.

1. Explain briefly why equal areas are swept in equal times even when the orbit is open. You may refer to the geometric argument in Lecture 6.
2. Using the velocity-circle construction from class, state the conditions on w and V_c for an ellipse, a parabola, and a hyperbola. Sketch the velocity-circle configurations for the parabola and hyperbola, marking O and C . Identify the limiting case between a bound orbit and a hyperbolic flyby.

Problem 2: A repulsive inverse-square force

In an idealized model of Rutherford scattering, an α particle of mass m approaches a heavy nucleus, which is held fixed at O . The force on the particle is

$$\mathbf{F} = \frac{\kappa}{r^2} \hat{\mathbf{r}}, \quad \kappa > 0,$$

where $\mathbf{r}$ points from the nucleus to the particle. Far from the nucleus, the incoming speed is $v_\infty > 0$ and the impact parameter is $b > 0$. The impact parameter is the perpendicular distance from O to the incoming straight-line asymptote.

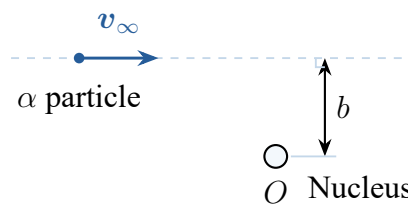


Fig. 1: The incoming asymptote and impact parameter.

1. Explain why the equal-area law remains valid for this repulsive force.
2. Adapt the geometric method from Lecture 6 to show that the particle follows one branch of a hyperbola, with the nucleus at a focus. Use a labeled diagram to show the essential construction. Calculus is not required.

Hint: For equal small changes in polar angle, compare the magnitudes and directions of the velocity changes with those in the attractive-force problem. The velocity endpoints again lie on a circle; the direction of the force is reversed.

Problem 3: The spatial pattern of the lunar tidal force

Let the Moon have mass M , and let its position relative to Earth's center O be $D\mathbf{n}$, where $\mathbf{n}$ is a unit vector pointing toward the Moon. A test mass m is at position $\mathbf{r}$ relative to O .

Use a translating, nonrotating frame whose origin follows Earth's center. Consider only the Moon's contribution to the relative acceleration. Assume $r = |\mathbf{r}| \ll D$.

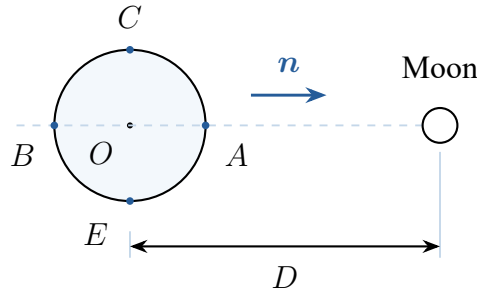


Fig. 2: Four points on Earth's surface. A is nearest the Moon and B is farthest from it. The diagram is not to scale.

1. Subtract the lunar gravitational acceleration of Earth's center from that of the test mass. Show that the lunar tidal force is

$$\mathbf{F}_{\text{tide}} = GMm \left[\frac{D\mathbf{n} - \mathbf{r}}{|D\mathbf{n} - \mathbf{r}|^3} - \frac{\mathbf{n}}{D^2} \right].$$

2. Expand to first order in r/D and show that

$$\mathbf{F}_{\text{tide}} \simeq \frac{GMm}{D^3} [3\mathbf{n}(\mathbf{n} \cdot \mathbf{r}) - \mathbf{r}].$$

Hint: $(1 + s)^{-3/2} \simeq 1 - \frac{3}{2}s$ for $|s| \ll 1$.

3. Let Earth's radius be R . Find the force vectors at $\mathbf{r} = \pm R\mathbf{n}$ and at surface points perpendicular to $\mathbf{n}$. Add force arrows at A , B , C , and E in Fig. 2. Explain why the tidal force stretches along the Earth–Moon line and compresses in transverse directions, producing two opposite tidal bulges in the idealized picture.

Problem 4: Comparing solar and lunar tides

Use the following approximate values:

$$M_{\odot} = 2.0 \times 10^{30} \text{ kg}, \quad D_{\odot} = 1.5 \times 10^{11} \text{ m},$$
$$M_{\text{Moon}} = 7.3 \times 10^{22} \text{ kg}, \quad D_{\text{Moon}} = 3.8 \times 10^8 \text{ m}.$$

Here $D_{\odot}$ and D_{Moon} are the distances from Earth's center to the Sun and Moon. Treat both as point masses and use the linear tidal approximation from Problem 3.

1. Calculate the ratio of the magnitudes of the solar and lunar gravitational accelerations at Earth's center.
2. For each body separately, consider a point on Earth's surface along the line toward that body. Calculate the ratio of the solar and lunar tidal acceleration magnitudes at these respective points. Which body produces the larger tidal effect? Explain why the answer differs from the comparison in part 1.
3. In an idealized equilibrium picture, explain how solar and lunar tides combine at new moon, full moon, and quarter moon. Draw simple diagrams of the relative positions of the three bodies. Identify the configurations associated with spring tides and neap tides.