HW13: Due on Dec 19. CODE NUMBER: ———— SCORE: ————

Problem #1: Statistical Mechanics

(a) In the canonical ensemble, show that the fluctuations in energy $\Delta E^2 = \langle E^2 \rangle - \langle E \rangle^2$ are proportional to the heat capacity.

(b) Show that in the canonical ensemble, the Gibbs entropy can be written as

$$S = k_B \frac{\partial}{\partial T}(T \log Z)$$

Problem #2: Entropy of a two-state system

1) What is the statistical (Boltzmann's) definition of entropy?

2) Using the two-state system as an example, explain why negative temperatures should be hotter than positive ones. You may use diagrams to help with the explanation.

Problem #3: Thermal conductivity [double point]

Joule conducted an experiment on an ideal gas composed of N molecules, initially held at a temperature T within a thermally isolated container. Initially, the gas occupied a volume V_1, while an adjacent volume V_2 was empty, separated by a removable partition. When the partition was suddenly removed, the gas expanded freely into the combined volume V_1+V_2. Despite this expansion, it was experimentally observed that the final temperature T_f remained equal to the initial temperature T, regardless of the particular values of T, V_1, or V_2.

1. Based on these observations, what can we deduce about the dependence of the internal energy U(T,V) of an ideal gas on its temperature T and volume V?
2. Starting again from the same initial configuration, suppose now that the system is kept in thermal contact with a reservoir at temperature T. If we then gradually move the partition to achieve the same final volume V_1+V_2, determine the change in the entropy of the ideal gas between the initial and final states. (Use the ideal gas law and include Boltzmann's constant k in your expressions.)
3. Now assume that the heat capacity at constant volume is given by $C_V=N\kappa T^\gamma$, where $\gamma>0$ and κ is a constant ensuring proper units. Repeating the process of slowly adjusting the partition to reach the final volume V_1+V_2, but this time with the system thermally isolated, find the final temperature of the gas.