Problem 1: Feynman's lectures on Physics

Read the first three chapters of Feynman's lectures on Physics. Write an essay with a few paragraphs about your thoughts on these lectures. For example, you might discuss in your essay which parts touch or impress you most in the chapters that you have read, or things that you didn't know before reading this note.

Problem 2: Vector analysis

1. $\vec{r}_1$ and $\vec{r}_2$ are arbitrary vectors in the 3D space. Try to verify that $\vec{r}_1 \cdot \vec{r}_2$ is invariant under rotations around the x, y, z axes, respectively. If you can prove it for arbitrary rotations, you can get extra points.
2. Verify that $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$. Using this, try to verify that the Lorentz force does not do any work.
3. Verify that $(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\, \vec{b} - (\vec{b} \cdot \vec{c})\, \vec{a}$.
4. Verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = (\vec{a} \times \vec{b} \cdot \vec{d})\, \vec{c} - (\vec{a} \times \vec{b} \cdot \vec{c})\, \vec{d}$

Problem 3: Polar vector v.s. axial vector

The usual vectors like displacement, velocity, and acceleration are called polar vectors. The angular momentum of a particle is defined as the cross-product between two polar vectors

$$\vec{L} = m\vec{r} \times \vec{v}, \tag{1}$$

which is called the axial vector.

Under spatial rotation, an axial vector transforms in the same way as a polar vector. Nevertheless, they behave differently under the mirror reflection. Please find out what they look like in a mirror.

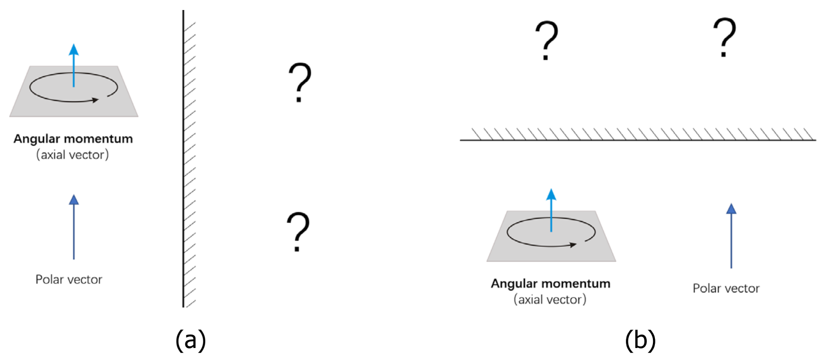

Figure 1: Polar and axial vectors under the mirror reflections. a) The mirror is parallel to the vectors; b) the mirror is perpendicular to the vectors.

Problem 4: Can a projectile reach its maximum height in a finite amount of time or not?

When you throw your coin up, you will observe it go up and fall into your hands. But some ancient people were confused about it. They argued in the ascending part of the process, when the coin goes upper and upper, its velocity becomes slower and slower. Within the same distance of ΔL, the coin will spend more and more time. When the coin is approaching the peak, its velocity goes to zero. It is unclear whether it could reach its peak at all! The descending process is the time-reversal counterpart of the ascending one. It is unclear whether the coin will fall at all since it starts from zero velocity.

Please follow a similar analysis as we examine the Zeno paradox in class to show that the coin will fall into your hand.

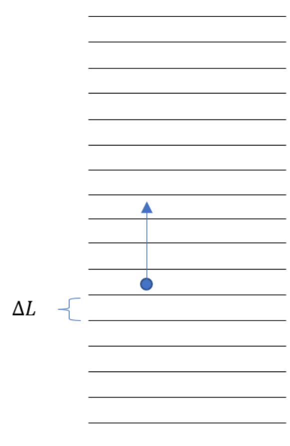

Figure 2: The decomposition of the acceleration vector

Problem 5: Tangential and normal accelerations

For a general motion along a curve, the natural coordinate system is more convenient. Around each point in the trajectory, we can use a small part of a circle to approximate the curve, such that the tangential direction and the curvature are maintained. We set the unit vector $\vec{e}_\tau$ along the tangent direction and that of $\vec{e}_n$ along the normal direction. As shown in the figure, the acceleration is decomposed into the tangential and normal components, accordingly, i.e.,

$$\vec{a} = \frac{dv}{dt}\vec{e}_\tau + \frac{v^2}{\rho}\vec{e}_n$$

where v is the velocity and ρ is the radius of curvature.

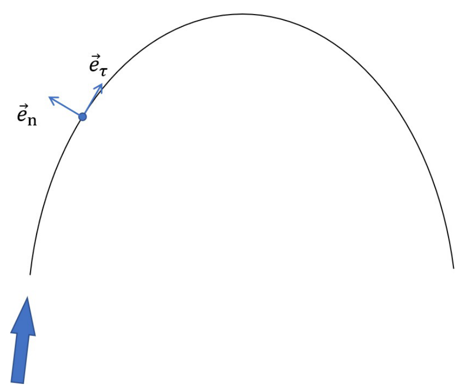

Figure 3: The tangential and normal accelerations.

1. The radius of the curvature can be formulated as

$$\frac{1}{\rho} = |\frac{d^2y}{dx^2}|/(1 + (\frac{dy}{dx})^2)^{3/2}$$

   Please verify that for a circle, it indeed gives the correct results.

2. Consider the projectile motion with an initial velocity $v_x = v_0 \cos\theta$ and $v_y = v_0 \sin\theta$. When the projectile reaches the maximal height, calculate the tangential and normal accelerations at this point. Confirm that your results are consistent with the gravity acceleration.