Jing

1. Given two vectors $\vec{A} = 2\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} - 8\hat{j} + 6\hat{k}$, evaluate the following expressions:

i	$\vec{A}\cdot\vec{B}=$	$2 - 32 = -30$
	$\vec{A}\times\vec{B}=$	$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 1 & -8 & 6 \end{vmatrix} = 24\hat{i} - 12\hat{j} - 20\hat{k}$

2. Find the derivatives of the following functions:

i	$f(x) = 5x^4 - 2x^3 + 7$	$f'(x) = 20x^3 - 6x^2$
ii	$g(z) = z \ln z$	$g'(z) = \ln z + 1$
iii	$h(r) = \sin r^3$	$h'(r) = (\cos r^3)\cdot 3r^2 = 3r^2 \cos r^3$

3. For $f(x) = x^3 - 6x^2 + 9x + 2$, find all local extrema by using the first derivative test. Find all the maxima and minima with the second derivative.

$f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x = 3, 1 \therefore$ local extrema

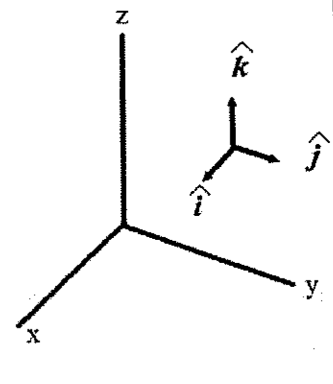
$f''(x) = 6x - 12 \quad x = 1 \Rightarrow f''(x) = -6 < 0$ maxima

$x = 3 \Rightarrow f''(x) = 6 > 0$ minima.

4. Compute the following integral:

$\int 2\sin(x) - 3\cos(x)\,dx =$	$-2\cos(x) - 3\sin(x)$.
$\int \frac{x\,dx}{3 - 2x^2} =$	$\int \frac{d(\frac{1}{2}x^2)}{3-2x^2} \overset{t=x^2}{=} \frac{1}{2}\int \frac{dt}{3-2t} = -\frac{1}{4}\int \frac{d(3-2t)}{(3-2t)} = -\frac{1}{4}\ln(3-2x^2)$
$\int \sin^2 x \cos x\,dx =$	$\int \sin^2 x\, d\sin x \underset{t=\sin x}{=} \int t^2\,dt = \frac{1}{3}\sin^3 x$

5. In the Cartesian coordinates, we generally use the ($\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$) to represent the three unit vector of a vector. What are the unit-vectors in the cylindrical coordinates and spherical coordinates? Along which direction are they pointing? Use diagram to show your answer.

<table>
<tr><th>Cartesian coordinates</th><th>Cylindrical coordinates</th><th>Spherical coordinates</th></tr>
<tr><td>Unit Vectors:
$\hat{\imath}, \hat{\jmath}, \hat{k}$
$\hat{e}_x, \hat{e}_y, \hat{e}_z$</td><td>Unit Vectors:
$\hat{r}, \hat{\phi}, \hat{z}$</td><td>Unit Vectors:
$\hat{r}, \hat{\phi}, \hat{\theta}$</td></tr>
<tr><td>Graph:

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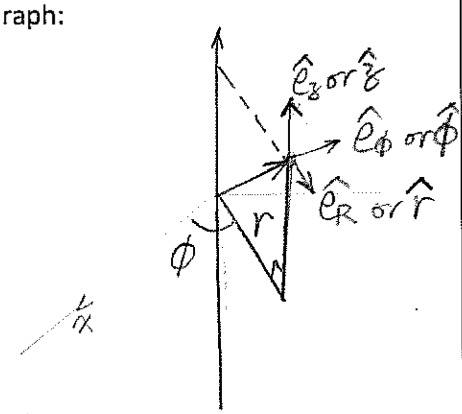
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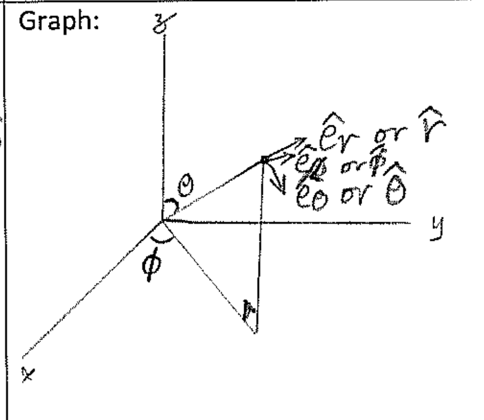
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6. Find the solutions for the differential equation: 6 x"+ 5 x = 0. Here x" represents the second derivative of x.

$$ax'' + bx = 0$$

$$ax'' = -bx$$

$$x'' = x \cdot \left(-\frac{b}{a}\right)$$

$$x = C \sin Et + D \cos Et$$

$$x' = CE \cos Et - DE \sin Et$$

$$x'' = -CE^2 \sin Et - DE^2 \cos Et$$

$$\therefore \quad -E^2 (C \sin Et + D \cos Et) = (C \sin Et + D \cos Et)(-E^2)$$

$$\Rightarrow \quad E^2 = \frac{b}{a}$$

$$E = \frac{\sqrt{5}}{\sqrt{6}}$$

$$\therefore \quad x = C \sin \sqrt{\frac{5}{6}}\, t + D \cos \sqrt{\frac{5}{6}}\, t$$