

Jing.

1. Given two vectors $\vec{A} = 2\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} - 8\hat{j} + 6\hat{k}$, evaluate the following expressions:

i	$\vec{A} \cdot \vec{B} =$	$2 \begin{vmatrix} 2 & 4 \\ 1 & -8 \end{vmatrix} = -30$
	$\vec{A} \times \vec{B} =$	$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 1 & -8 & 6 \end{vmatrix} = 24\hat{i} - 12\hat{j} - 20\hat{k}$

2. Find the derivatives of the following functions:

i	$f(x) = 5x^4 - 2x^3 + 7$	$f'(x) = 20x^3 - 6x^2$
ii	$g(z) = z \ln z$	$g'(z) = \ln z + 1$
iii	$h(r) = \sin r^3$	$h'(r) = (\cos r^3) \cdot 3r^2 = 3r^2 \cos r^3$

3. For $f(x) = x^3 - 6x^2 + 9x + 2$, find all local extrema by using the first derivative test. Find all the maxima and minima with the second derivative.

$$f'(x) = 3x^2 - 12x + 9 = 0 \Rightarrow x = 3, 1 \text{ local extrema}$$

$$f''(x) = 6x - 12 \Rightarrow x = 1 \Rightarrow f''(x) = -6 < 0 \text{ maxima}$$

$$x = 3 \Rightarrow f''(x) = 6 > 0 \text{ minima}$$

4. Compute the following integral:

$\int 2 \sin(x) - 3 \cos(x) dx =$	$-2 \cos(x) - 3 \sin(x)$
$\int \frac{x dx}{3-2x^2} =$	$\int \frac{d(\frac{1}{2}x^2)}{3-2x^2} = \frac{1}{2} \int \frac{dt}{3-2t} = -\frac{1}{4} \int \frac{d(3-2t)}{(3-2t)} = -\frac{1}{4} \ln(3-2x^2)$
$\int \sin^2 x \cos x dx =$	$\int \sin^2 x d \sin x = \int t^2 dt = \frac{1}{3} \sin^3 x$

5. In the Cartesian coordinates, we generally use the $(\hat{i}, \hat{j}, \hat{k})$ to represent the three unit vector of a vector. What are the unit-vectors in the cylindrical coordinates and spherical coordinates? Along which direction are they pointing? Use diagram to show your answer.

Cartesian coordinates	Cylindrical coordinates	Spherical coordinates
Unit Vectors: $\hat{i}, \hat{j}, \hat{k}$ $\hat{e}_x, \hat{e}_y, \hat{e}_z$	Unit Vectors: $\hat{r}, \hat{\phi}, \hat{z}$	Unit Vectors: $\hat{r}, \hat{\theta}, \hat{\phi}$
Graph:	Graph:	Graph:

The diagram for Cartesian coordinates shows a 3D Cartesian system with axes x, y, z and unit vectors \hat{i} , \hat{j} , \hat{k} . The diagram for cylindrical coordinates shows a 2D plane with radial axis \hat{r} at angle ϕ from the x-axis, and vertical axis \hat{z} . The diagram for spherical coordinates shows a 3D space with radial axis \hat{r} at angle θ from the z-axis, polar angle ϕ from the x-axis, and azimuthal angle ψ from the vertical \hat{z} .

6. Find the solutions for the differential equation: $6x'' + 5x = 0$. Here x'' represents the second derivative of x .

$$ax'' + bx = 0$$

$$ax'' = -bx$$

$$x'' = x \cdot \left(-\frac{b}{a}\right)$$

$$x = C \sin Et + D \cos Et$$

$$x' = CE \cos Et - DE \sin Et$$

$$x'' = -CE^2 \sin Et - DE^2 \cos Et$$

$$\therefore -E^2 (C \sin Et + D \cos Et) = (C \sin Et + D \cos Et)(-E^2)$$

$$\Rightarrow E^2 = \frac{b}{a}$$

$$E = \sqrt{\frac{5}{6}}$$

$$\therefore x = C \sin \sqrt{\frac{5}{6}} t + D \cos \sqrt{\frac{5}{6}} t$$