General Physics I

Lect16. Temperature and Distribution

Based on Feynman Lectures Ch.39&40

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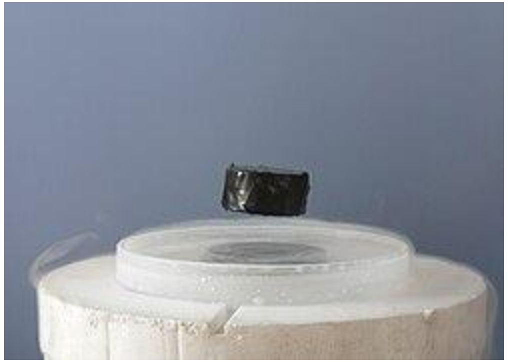
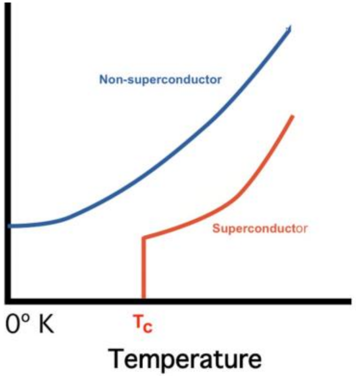
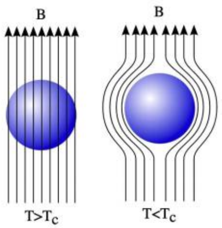
Temperature is everywhere







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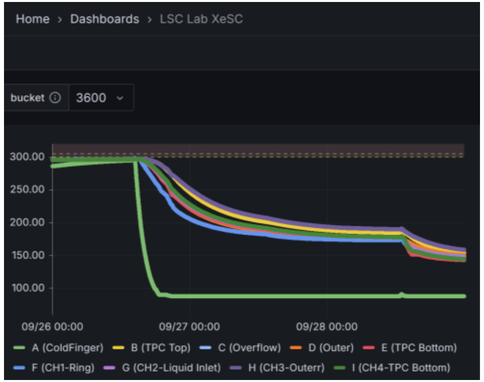

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几分钟前更新

22°C 晴朗

高温22° 低温14°

今天	周一 17	周二 18	周三 19	周四 20	周五 21
22° 14°	14° 5°	10° 2°	12° 1°	15° 3°	18° 5°

2 上午	5 上午	8 上午	11 上午	2 下午	5 下午	8 下午	11 下午
14°	15°	14°	12°	10°	9°	7°	6°
		2%	34%	21%	19%	10%	4%

We can establish temperature scales using different reference points. Like **melting/boiling points** of pure water at atmospheric pressure. But **Kelvin** is probably the most natural one in physics.

But what is temperature exactly?

Temperature

$$<m_1\vec{v}_1^{\,2}>=<m_2\vec{v}_2^{\,2}>$$

- For two different gases, their average kinetic energy is the **same in equilibrium**----the heavy ones will move slower than the light ones. We see that, not just for gas, this quantity seems to agree for any kind of matter.
- We use the word "**temperature**" to denote the mean molecular kinetic energy of gas, but it is rather the property of itself rather than just gas.
- There is a simple linear relation between mean kinetic energy and temperature T, for each d.o.f.

$$\frac{1}{2}k_B T$$

$k_B=1.38\times10^{-23}$ joule for every Kelvin, "Boltzmann constant"

Theorem 1: the mean value of the kinetic energy is kT/2 for each independent motion (d.o.f.)


Ludwig Boltzmann

Ludwig Eduard Boltzmann
(1844 – 1906)
Austrian physicist who developed the statistical approach to thermodynamics, showing how the collective behavior of atoms gives rise to macroscopic physical laws.

The Ideal Gas Law

- Recall from the last class, we have

$$PV = 2/3 \cdot N \langle \tfrac{1}{2}mv^2 \rangle$$

- Let's now plug in $< \frac{1}{2} m v_i^2 > = \frac{1}{2} k_B T$,

$$PV = Nk_BT \longrightarrow$$

- Beware, chemists count atoms in *moles*, one mole has $N_0=6.02\times10^{23}$ objects, and they define a number $R = N_0 k_B = 8.31$ J·mole^{-1}K^{-1}
- In chemistry/high school textbooks, while they also call it N, it's actually in the unit of *moles* and the idea gas law writes

$$PV = NRT$$

- "**Equation of state**" is an equation that relates **state variables**. If we know the rest of the variables (e.g.,V, N, T), we know the last variable (e.g., P)

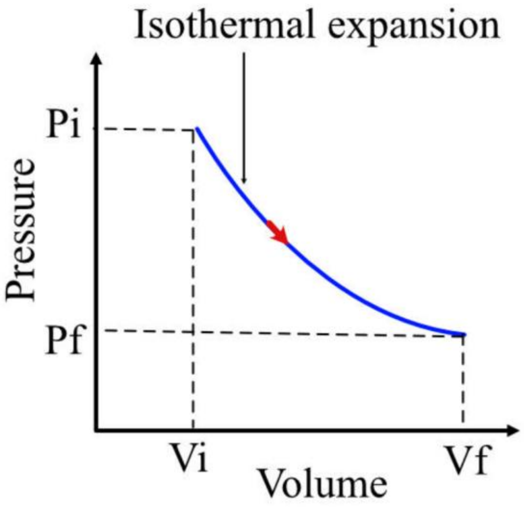
"So equal volumes of **different gases**, at the same pressure and temperature, have the same **number of molecules**, because of Newton's laws. That is an amazing conclusion!"

The opposite of state variable is called path-dependent variables. Like mechanical work done or heat transfer as a system moves from one state to another depends on the path taken between states
(see similarity with a *conservative force potential*)

Application: Ideal Gas Expansions

- Unlike **temperature** is a state, **expansion/compression** are processes, we need to specify how it is done, i.e., the “**path**”.
- **Isothermal** (等温) means *constant temperature*.

  T = const. → **PV** = const.
- Consider gas expanded slowly against a piston – the gas does work on the piston, so would cool, but heat is allowed to flow in from the surroundings to keep the gas at constant temperature



Isothermal processes are hyperbola on a ***P*−*V*** diagram.

$$\Delta U = 0 \quad \Rightarrow \quad \mathrm{d}Q = \mathrm{d}U - \text{đ}W = p\,dV = \frac{RT}{V}dV$$

$$\Rightarrow \quad \Delta Q = \int_{V_\mathrm{i}}^{V_\mathrm{f}} p\,dV = \int_{V_\mathrm{i}}^{V_\mathrm{f}} \frac{RT}{V}dV = RT\,ln\left(\frac{V_1}{V_0}\right)$$

Diatomic Gas

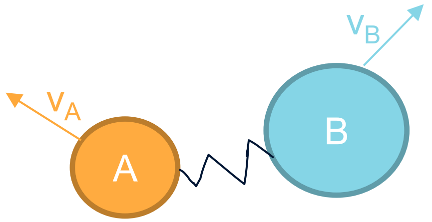
- What happens if there are **forces** present? Assume the gas molecule is composed of *two* different atoms, A and B, held by a force (chemical bond, spring). The center of mass velocity:

$$\vec{v}_{CM}=(m_A\vec{v}_A+m_B\vec{v}_B)/(m_A+m_B)=(m_A\vec{v}_A+m_B\vec{v}_B)/M$$

$$\vec{v}_{CM}^{\,2}=[m_A^2\vec{v}_A^{\,2}+m_B^2\vec{v}_B^{\,2}+(2m_A m_B)(\vec{v}_A\cdot\vec{v}_B)]/M^2$$

$$<\frac{1}{2}M\vec{v}_{CM}^{\,2}>=(m_A\frac{3}{2}kT+m_B\frac{3}{2}kT+m_A m_B<\vec{v}_A\cdot\vec{v}_B>)/M$$

$$=\frac{3}{2}kT+m_A m_B<\vec{v}_A\cdot\vec{v}_B>/M$$



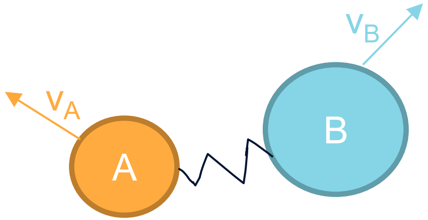
We will prove that $<\vec{v}_A\cdot\vec{v}_B>$ is indeed zero in the next slide..

Diatomic Gas (Cont’d)

- To prove that <$\vec{v}_A \cdot \vec{v}_B$> is zero, we do the same trick again $\langle \boldsymbol{w} \cdot \boldsymbol{v}_{\mathrm{CM}} \rangle = 0$

$$\boldsymbol{w} \cdot \boldsymbol{v}_{\mathrm{CM}} = \frac{(\boldsymbol{v}_A - \boldsymbol{v}_B) \cdot (m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B)}{M}$$

$$= \frac{m_A v_A^2 + (m_B - m_A)(\boldsymbol{v}_A \cdot \boldsymbol{v}_B) - m_B v_B^2}{M}$$



- Now that both A and B are in the equilibrium, $\langle m_A v_A^2 \rangle = \langle m_B v_B^2 \rangle$

$$(m_B - m_A)\langle \boldsymbol{v}_A \cdot \boldsymbol{v}_B \rangle = 0$$

- If the masses of A and B are different, we have $\langle \boldsymbol{v}_A \cdot \boldsymbol{v}_B \rangle = 0$, and the entire molecule can be regarded as a single molecule of mass M, s.t.

$$\left\langle \frac{1}{2} M v_{\mathrm{CM}}^2 \right\rangle = \frac{3}{2} kT$$

Total K.E. 3kT

$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

3kT/2 + 3kT/2

The average kinetic energy of the ***internal*** motions of the diatomic molecule is also **3kT/2**!

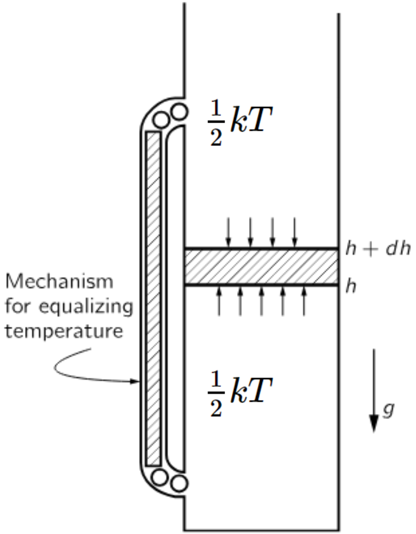
Statistical Mechanics - laws of mechanics for thermal equilibrium

- How are the molecules distributed in space (i.e., with a *force field*)?
- See right, we know that molecules at all heights should have the same temperature if we neglect the convection (wind), so what changed?
- Recall P=nkT, we can write the pressure difference of h and h+dh, dP=kTdn, which is equal to the weight of a small volume of air: $-mgn\,dh = kTdn$
- So,

$$\frac{dn}{dh} = -\frac{mg}{kT}n \Longrightarrow n = n_0 e^{-mgh/kT}$$

We find that the density goes down *exponentially* with height.

- For molecules of *different masses*, the exponential is different--heavier decrease with altitude faster.



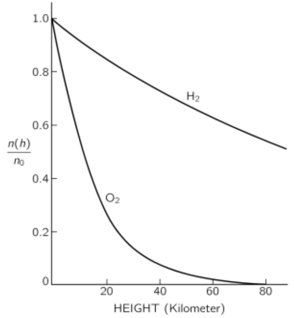


Fig. 40–2. The normalized density as a function of height in the earth's gravitational field for oxygen and for hydrogen, at constant temperature.

O_2 decrease by **50%** over 10km!

The Boltzmann's law

- Note such exponential relation can be applied to *any potential*. To generalize, assume the gas is subject to a force F in the x-direction, balanced by the pressure change s.t. $F n\, dx = dP = kT\, dn$, again we find

$$F = kT \frac{d}{dx}(\ln n)$$

- Notice that –Fdx is the work to take a molecule from x to x+dx, i.e., the change in potential energy (P.E.), s.t. $d(\ln n) = -d(\mathrm{P.E.})/kT$;
- General spatial distribution (density) of molecule in terms of potential energy (P.E.):

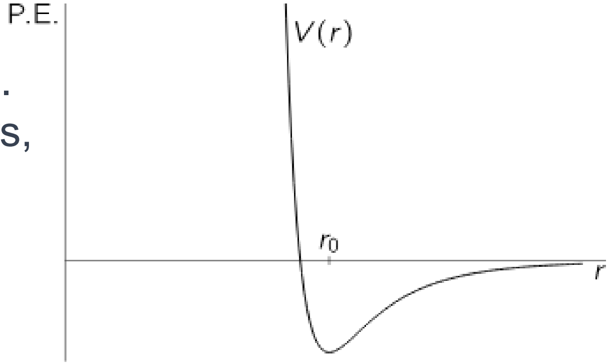
$$n = (\mathrm{constant})e^{-\mathrm{P.E.}/kT}$$

- **Boltzmann's law：** the probability of finding molecules in a given spatial arrangement varies *exponentially* with the negative of the *potential energy* of that arrangement, divided by kT.

Application: Evaporation of a liquid

- Suppose the potential energy function of each of two molecules (i and j) depends only on their separation, $V(r_{ij})$. The total potential energy would be the sum of all the pairs, in the full set of molecules. The probability of finding molecules in any particular combination is

$$\exp\left[-\sum_{i,j} V(r_{ij})/kT\right]$$



- For $|V(r_0)| \ll kT$, the exponential close to 1, the molecule can be anywhere. The potential does not make much difference for where the particle should be → **evaporation**

- For $|V(r_0)| \gg kT$, large positive exponent around r_0. In a given volume, molecules are likely to be at the distance of minimum energy than far apart → **condensation**

As the temperature falls, the atoms clump in lumps, and reduce to liquids, solids, and molecules, and as you heat them up they evaporate.

This is a *complicated* problem (due to a tremendous number of variables) sometimes referred to as the "**many-body problem**", or "**condensed matter physics**".

The Distribution of Molecular Speeds

- Now we go on to discuss the distribution of **velocities**. We start by counting how many molecules are passing from below to above the plane h=0 and h=h.

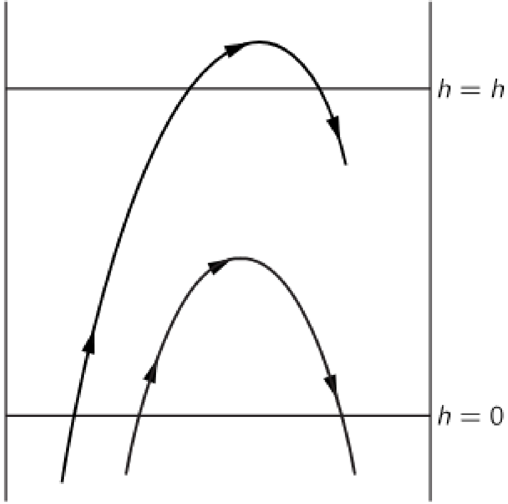
$$\begin{pmatrix}\text{Number passing } h=0\\ \text{with } v_z>u\end{pmatrix} = \begin{pmatrix}\text{number passing } h=h\\ \text{with } v_z>0\end{pmatrix} \qquad mu^2/2=mgh$$

- From the same temperature we know that the distribution of velocities is the same, we use $n=n_0e^{-mgh/kT}$ we find before and substitute with u, and find that

$$\frac{n_{>u}(0)}{n_{>0}(0)}=e^{-mgh/kT}=e^{-mu^2/2kT}$$

- Like the previous case, we can generalize the velocity to

$$n_{>u}\propto e^{-\text{kinetic energy}/kT}$$

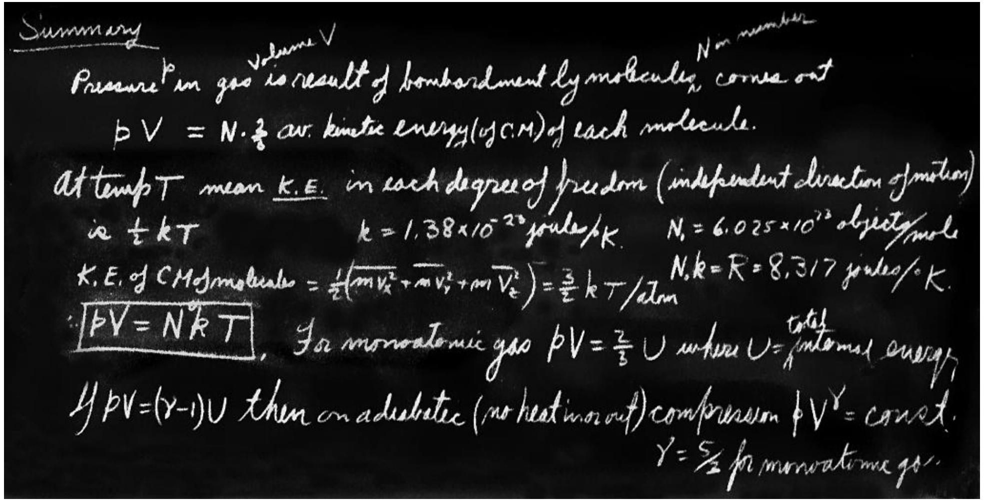


We will continue the discussion next lecture..

Chapter summary

- Temperature ↔ Kinetic energy
- Ideal Gas Law: **$PV = Nk_BT$**
- Application: Isothermal expansion
- Check: Diatomic Gas
- The Boltzmann's law
- Distribution of Molecular Speeds

Feynman Lectures Ch.39 Summary



Summary

Pressure p in gas (volume V) is result of bombardment by molecules (N = number), comes out

$pV = N\cdot\frac{2}{3}$ av. kinetic energy (of C.M.) of each molecule.

At temp T mean K.E. in each degree of freedom (independent direction of motion) is $\frac{1}{2}kT$ $k = 1.38\times10^{-23}$ joules/°K $N_0 = 6.025\times10^{23}$ objects/mole

K.E. of CM of molecules $= \frac{1}{2}(m\overline{v_x^2} + m\overline{v_y^2} + m\overline{v_z^2}) = \frac{3}{2}kT$/atom $N_0k = R = 8.317$ joules/°K.

$\therefore \boxed{pV = NkT}$. For monoatomic gas $pV = \frac{2}{3}U$ where U = total internal energy

If $pV = (\gamma-1)U$ then on adiabatic (no heat in or out) compression pV^γ = const.

$\gamma = 5/3$ for monoatomic gas.