General Physics I

Lect15. The Kinetic Theory of Gases

Based on Feynman Lectures Ch.39

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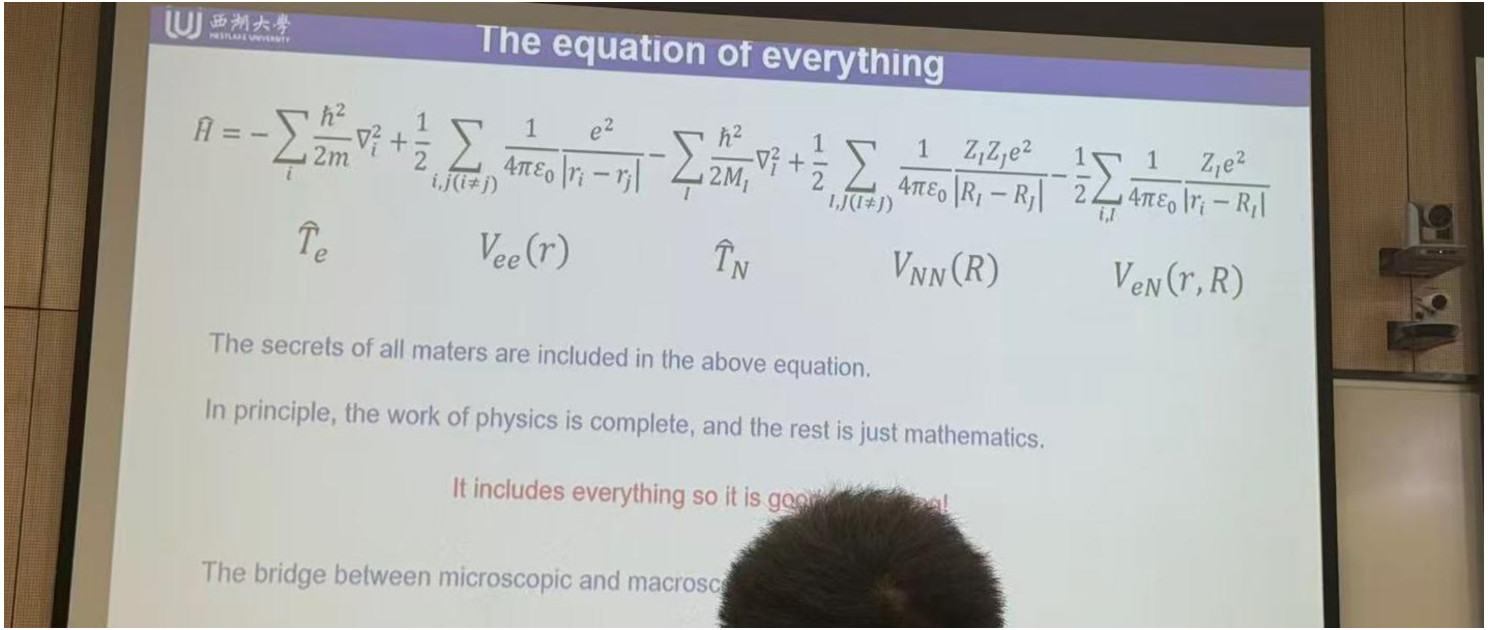
More is different! --- P.W. Anderson



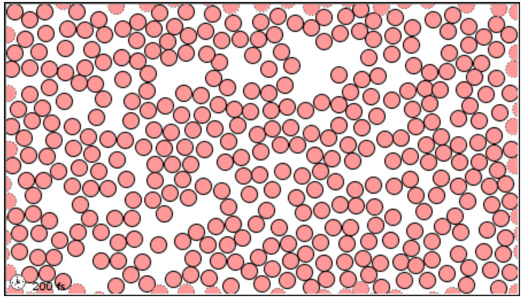
But everything is difficult…

The Study of "Matter"

•The analysis of **matter** from a physical perspective is complex, given that it's made of **numerous atoms** interacting electrically and mechanically.

•Unlike simpler systems like mechanics or light, where we started with **precise laws** (like Newton's), matter is too intricate to be understood directly from these laws alone.

•The study of matter requires understanding the **probabilistic behavior** of many atoms (macroscopic). In history, people started with imprecise classical statistical mechanics but progressively refined thermodynamics with quantum mechanics.



Physics is the Art of Approximation

近似

The world is too complex to be described exactly: we need **approximations.**

- Real-world analysis of matter starts with **physical intuition** and **appropriate approximations**, rather than direct mathematical solutions from fundamental equations.

- The discussion begins with **gases** and will extend to their **properties** like pressure, volume, and temperature—many of the rules are empirical at first.

- We will understand:

Why gas at the same temperature and pressure has the same number of molecules? Why does gas heat up as it is compressed, or expand as it is heated? And so on…

Gas Pressure

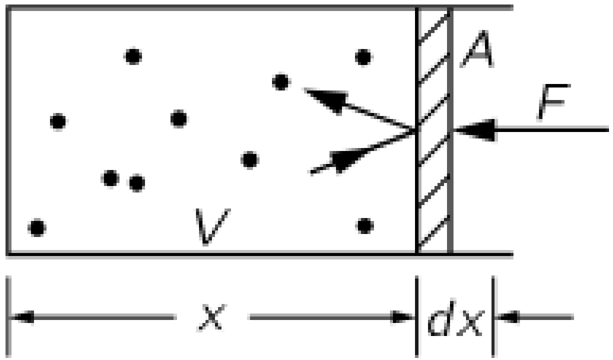
•Gas molecules in constant motion exert a force on surfaces (e.g., eardrums). This is perceived as **pressure**.

$$P = F/A$$

•Consider a box of volume V, with atoms moving around inside the box with various velocities they bang against the piston. The piston is assumed to be a perfect reflector (no heating up).

$$dW = F(-dx) = -PAdx = -PdV$$

•F is the force needed to balance the banging of the molecules. The minus sign as we compress it, we decrease the volume.



Gas Pressure

- How many atoms are hitting? The small volume dV is occupied by the atoms to hit the piston

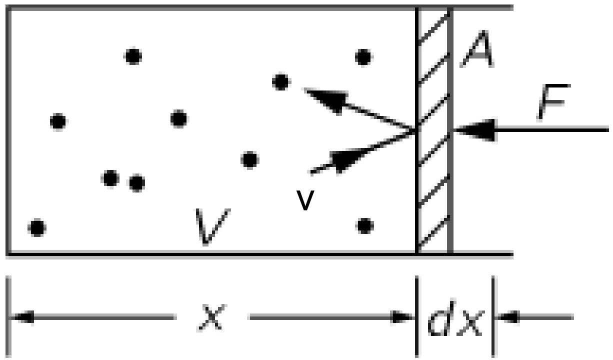
$$dV = v_x A\ dt$$

- A perfectly elastic collision with a piston doubles the momentum transfer -> $2mv_x$
- The number of the molecule with velocity v is defined as n(v), then the **total momentum** transfer per time:

$$dP = 2mv_x \cdot n(v) \cdot v_x A\ dt$$

- Recall that dP = F dt, and half of the v_x is pointing away from the piston -> ½, we have

$$F = mv_x^2 \cdot n(v) \cdot A$$



Gas Pressure

- Denote the velocity distribution n(v) as the product of average velocity $\langle v_x^2 \rangle$ and volume density n = N/V

$$F = mv_x^2 \cdot n(v) \cdot A$$
$$= n \cdot m \cdot A \langle v_x^2 \rangle$$

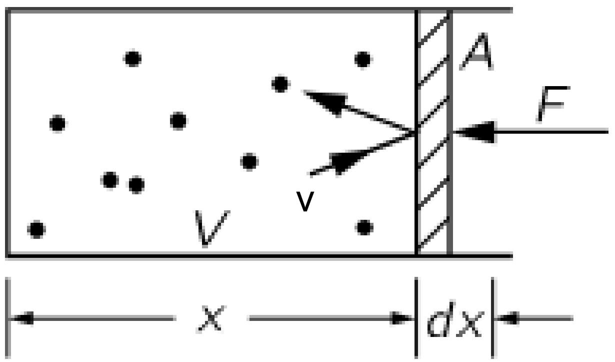
- Recall P = F/A, we have

$$P = n \cdot m \ \langle v_x^2 \rangle$$
$$= 2\, n \cdot \langle ½mv_x^2 + ½mv_y^2 + ½mv_z^2 \rangle / 3$$
$$= 2/3 \cdot n \ \langle ½mv^2 \rangle$$

- The equation of state

$$PV = 2/3 \cdot N \langle ½mv^2 \rangle = 2/3 \cdot U$$

U is the total **internal energy** of the monoatomic gas, where we disregard excitation or motion inside the atoms.



Notice that the number 2/3 changes with different types of gas. Conventionally, the general expression of the equation is written as

$$PV = (\gamma - 1)U$$

We know that for a monatomic gas like helium, γ=5/3

*γ=Cp/Cv, ratio of specific heats (homework)

Adiabatic 绝热 Compression

- Definition: A process where **no heat** is transferred to or from the system, and all work done on the gas changes its internal energy.
- For an adiabatic compression, all the work done goes into changing the internal energy

$$PdV = -dU$$

- Since $U = PV/(\gamma-1)$, and $PdV = -(PdV+VdP)/(\gamma-1)$

$$\Rightarrow dP/P+\gamma\, dV/V=0$$

$$\Rightarrow \gamma \ln V + \ln P = \text{const.}$$

$$\Rightarrow PV^{\gamma} = \text{const.}$$

- Under adiabatic conditions, the pressure times the volume to the $\gamma=5/3$ power is a constant for a monatomic gas -> experimentally confirmed

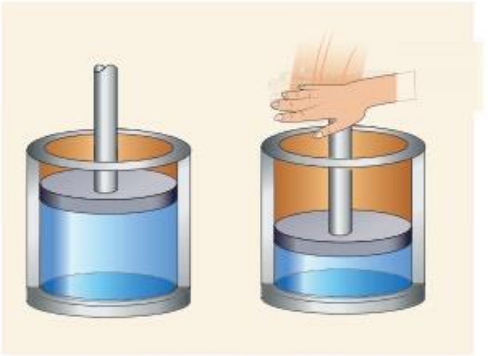


Figure 8.29 (b): When the gas is compressed or expanded so fast, the gas cannot exchange heat with surrounding even though there is no thermal insulation.

Extension: Stellar Photon Gas

- In astrophysics, the gas of **photons** can be used to describe the features of very hot stars, where atomic contribution can be neglected.

$$P = nm\langle v_x v_x\rangle = n\langle p_x v_x\rangle = n/3 \langle \vec{\mathbf{p}} \cdot \vec{\mathbf{v}} \rangle$$

- Photon energy $E=pc=\langle \vec{p} \cdot \vec{v} \rangle$, and total internal energy U=NE=nVE, hence for photon gas

$$PV = n/3\ E\ V = U/3$$

- This means γ -1 = 1/3, such that γ = 4/3, and we have the **compressibility of radiation**!

$$PV^{4/3} = \text{const.}$$

- As the volume decreases due to gravitational pull, the pressure must increase to maintain equilibrium.

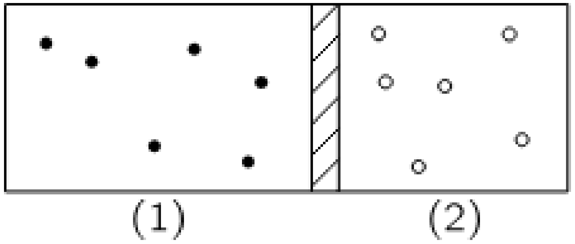
Understanding photon gas behavior is essential for studying the life cycle of stars, especially during phases like white dwarf and neutron star formation, where temperature and pressure are extreme.

Kinetic Energy: Two Gases in a Box with Piston

- In container (1) the atoms have mass m_1, velocity v_1, and there are n_1 per unit volume. Same for (2). What are the conditions for equilibrium?
- Recall $P=(2/3)n\langle mv^2/2\rangle$, so

$$n_1\langle m_1 v^2_1/2\rangle = n_2\langle m_2 v^2_2/2\rangle$$

- The energy of the two boxes will come to an **equilibrium** temperature eventually, as the piston wiggles and keeps the pressure on two sides equal----**heat conduction**!
- The system comes to an **equilibrium** where it picks up energy from the atoms at about the same rate as it puts energy back into them, for both sides.



Kinetic Energy: Two Gases Mixed

- We have **two molecules** of different mass colliding and the collision is viewed in the center-of-mass frame.

$$\vec{v}_{CM}=(m_1\vec{v}_1+m_2\vec{v}_2)/(m_1+m_2)$$

- From **conservation**, we know the relative velocity $\vec{\mathbf{w}}=\vec{\mathbf{v}}_1-\vec{\mathbf{v}}_2$ is changed, s.t. $|\vec{w}|=|\overrightarrow{w'}|$ but in different directions
- We know at **equilibrium**, $\vec{\mathbf{w}}$ will not have a preference direction as the collisions are totally randomized, hence

$$\langle\vec{\mathbf{v}}_{CM}\cdot\vec{\mathbf{w}}\rangle=0$$

Expand $\vec{\mathbf{w}}\cdot\vec{\mathbf{v}}_{CM}=(\vec{\mathbf{v}}_1-\vec{\mathbf{v}}_2)(m_1\vec{\mathbf{v}}_1+m_2\vec{\mathbf{v}}_2)/(m_1+m_2)$

$=[(m_1\vec{\mathbf{v}}_1^2-m_2\vec{\mathbf{v}}_2^2+(m_2-m_1)(\vec{\mathbf{v}}_1\cdot\vec{\mathbf{v}}_2)]/(m_1+m_2)$

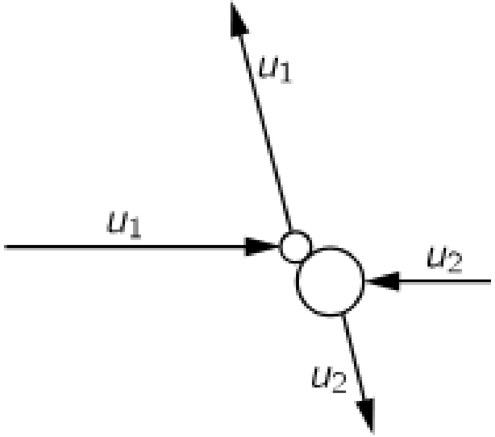


Fig. 39–3. A collision between unequal atoms, viewed in the CM system. $u_1=|\mathbf{v}_1-\mathbf{v}_{CM}|$, $u_2=|\mathbf{v}_2-\mathbf{v}_{CM}|$.

$\langle\vec{v}_1\cdot\vec{v}_2\rangle=0$ particles move at random (uncorrelated) directions, hence

$$\langle m_1\vec{v}_1^2\rangle=\langle m_2\vec{v}_2^2\rangle$$

Same kinetic energy!